ADT for **Quick Search Tree** data structure

Lecture 17

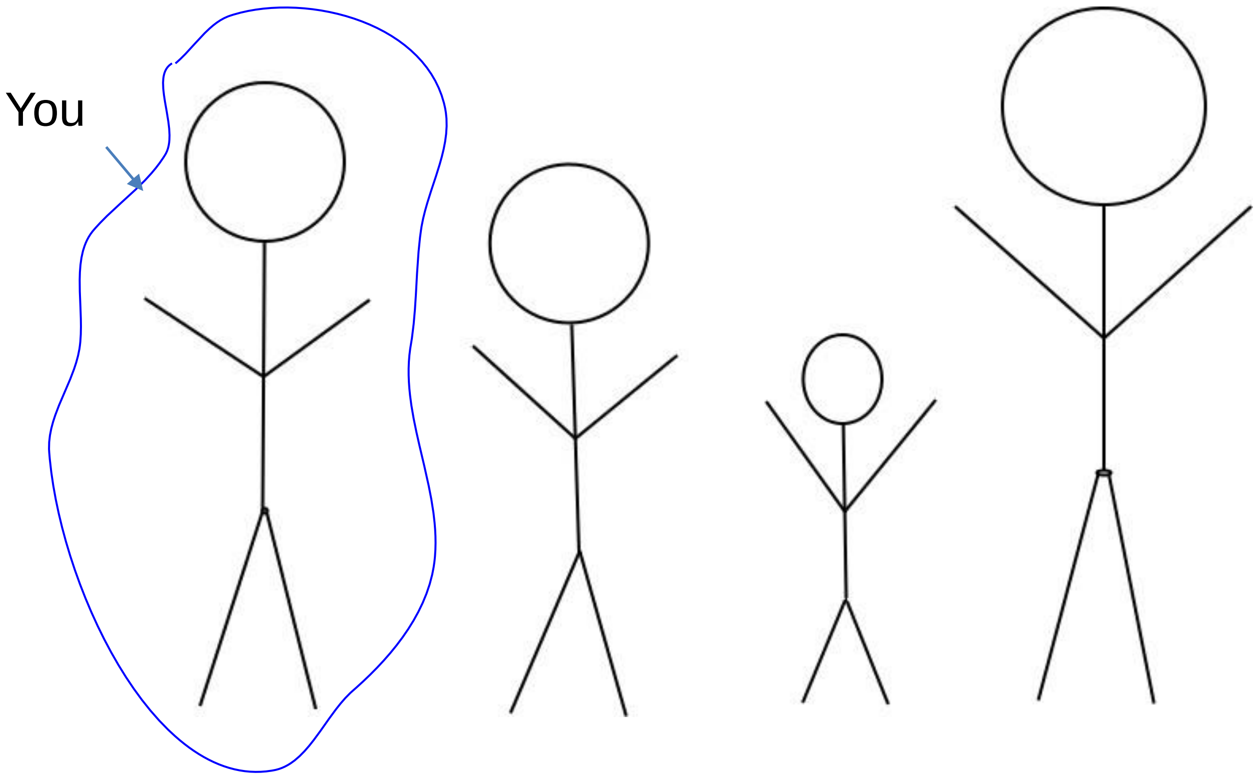
by Marina Barsky

Motivation 1: Searching

Find company address in the address book

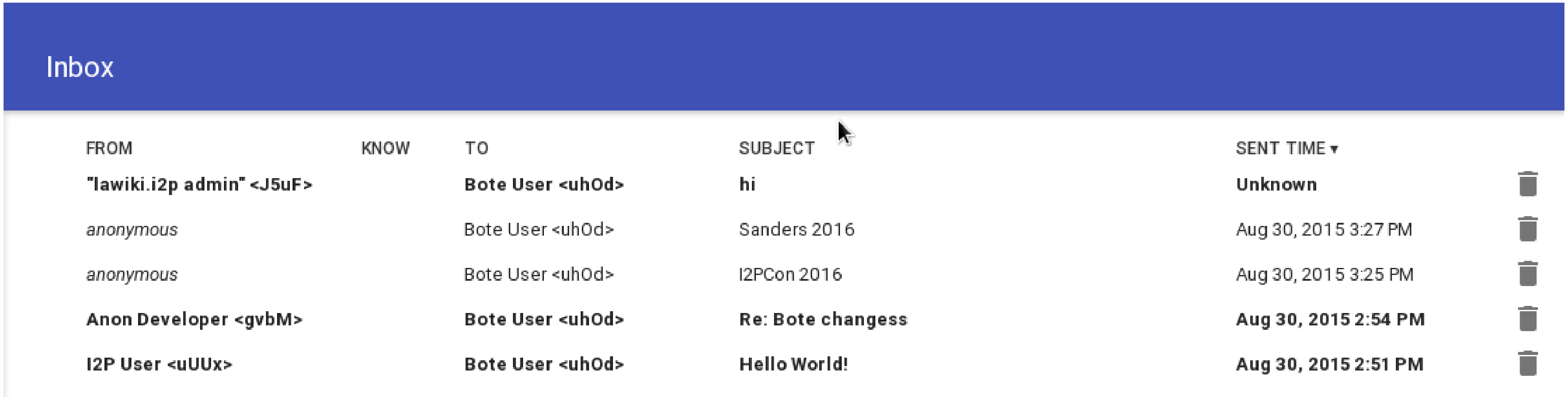
Motivation 2: Closest Height

Find people in your class whose height is closest to yours.



Motivation 3: Date Ranges

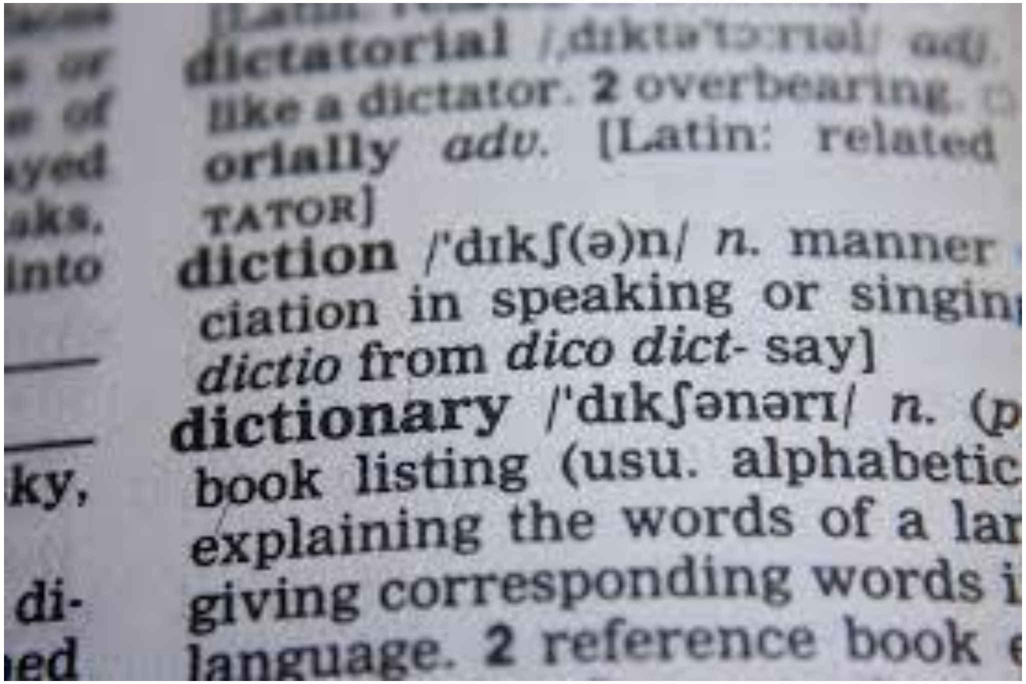
Find all emails received in a given period



FROM	KNOW	TO	SUBJECT	SENT TIME ▾
"lawiki.i2p admin" <J5uF>		**Bote User <uhOd>**	**hi**	**Unknown**
anonymous		Bote User <uhOd>	Sanders 2016	Aug 30, 2015 3:27 PM
anonymous		Bote User <uhOd>	I2PCon 2016	Aug 30, 2015 3:25 PM
Anon Developer <gvbM>		**Bote User <uhOd>**	**Re: Bote changess**	**Aug 30, 2015 2:54 PM**
I2P User <uUUx>		**Bote User <uhOd>**	**Hello World!**	**Aug 30, 2015 2:51 PM**

Motivation 4: Partial Search

Find all words that **start with** some given *prefix*

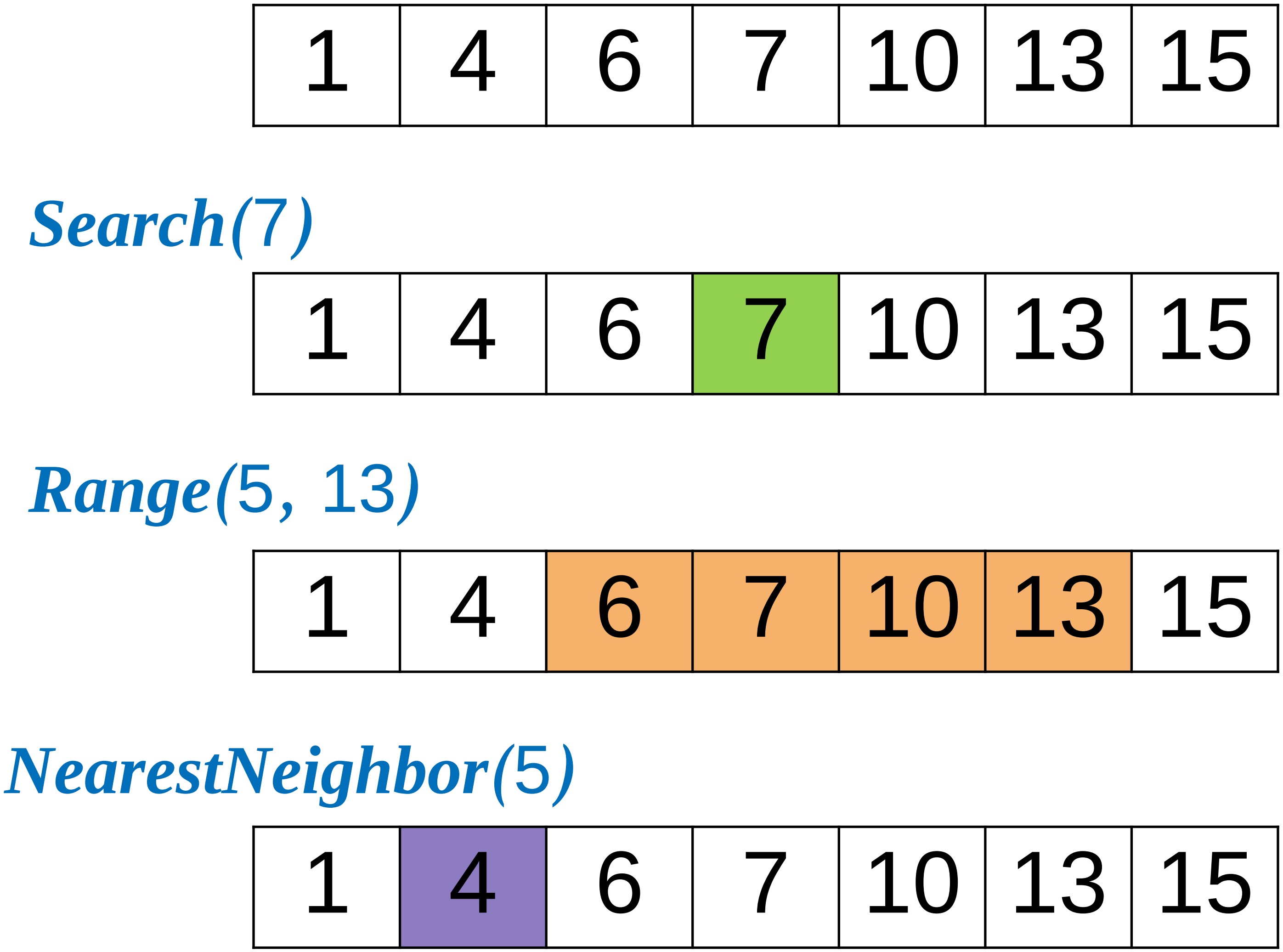


dictatorial /ˌdɪktəˈtɔːrɪəl/ *adj.*
like a dictator. 2 overbearing.
orially *adv.* [Latin: related
TATOR]
diction /ˈdɪkʃ(ə)n/ *n.* manner
ciation in speaking or singin
dictio from *dico dict-* say]
dictionary /ˈdɪkʃənərɪ/ *n.* (
book listing (usu. alphabetic
explaining the words of a la
giving corresponding words
language. 2 reference book

Specification

A ***Quick Search ADT*** stores a number of elements each with a *key* and supports the following operations:

- ***Search***(x): returns the element with the key=x
- ***Range***(lo, hi): returns all elements with keys between lo and hi
- ***NearestNeighbor***(x): returns *an element* with the key closest to x



1	4	6	7	10	13	15

***Search**(7)*

1	4	6	7	10	13	15

***Range**(5, 13)*

1	4	6	7	10	13	15

***NearestNeighbor**(5)*

1	4	6	7	10	13	15

Sorted keys

1	4	6	7	10	13	15

- Is seems that the best idea is to store the elements **sorted** by keys

How to make this dynamic?

- Store keys in **sorted order**
- But we also want to be able to add/remove keys efficiently

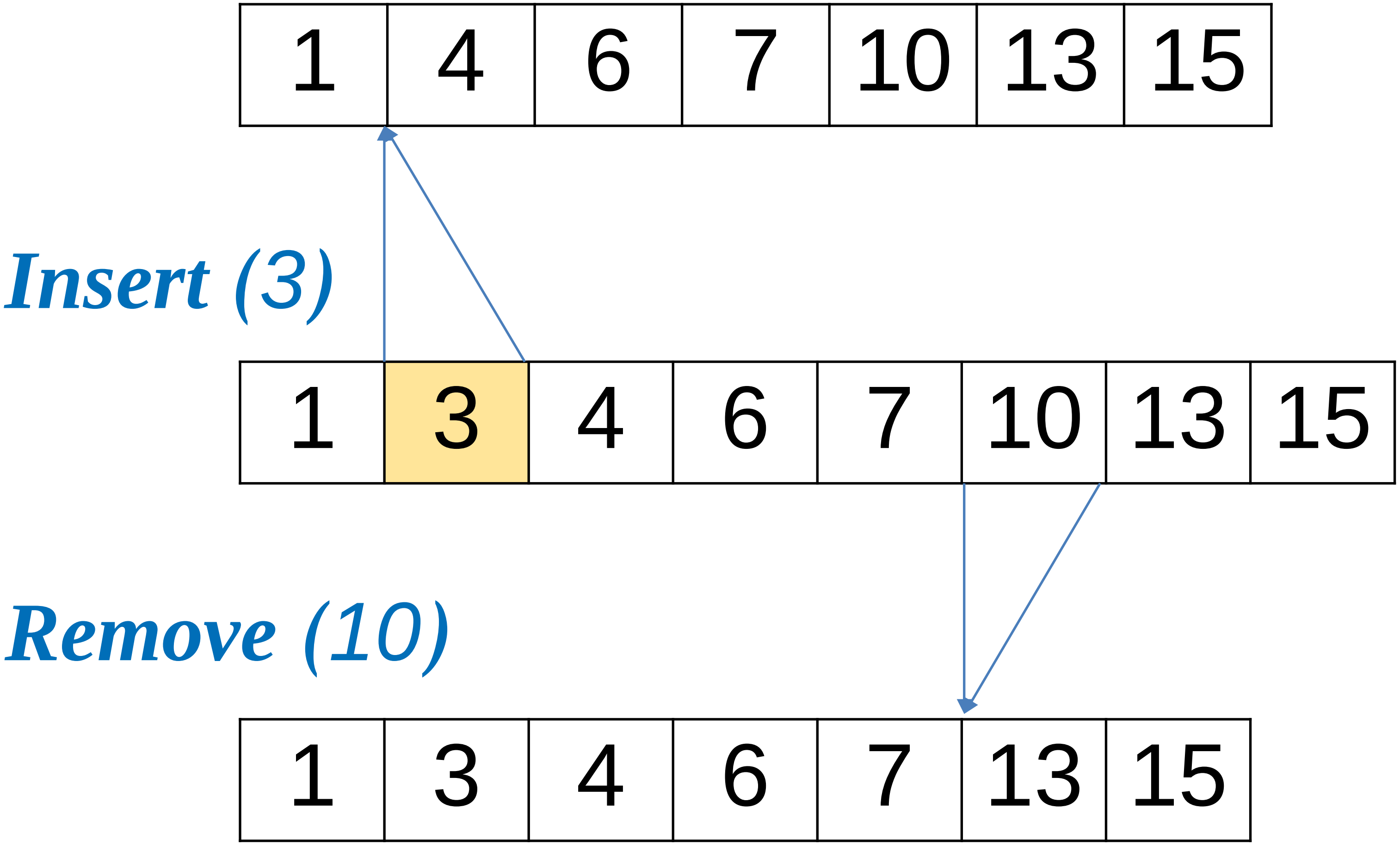
Quick Search ADT

Full Specification

A ***Quick Search ADT*** stores a number of elements each with a *key* and supports the following operations:

- ➔ ***Search(x)***: returns the element with the key=x
- ➔ ***Range(lo, hi)***: returns all elements with keys between *lo* and *hi*
- ➔ ***NearestNeighbor(x)***: returns an element with the key closest to x
- ➔ ***Insert(x)***: adds an element with key x
- ➔ ***Remove(x)***: removes the element with key x

Example



1	4	6	7	10	13	15

Insert (3)

1	3	4	6	7	10	13	15

Remove (10)

1	3	4	6	7	13	15

Possible Implementations

1	4	6	7	10	13	15

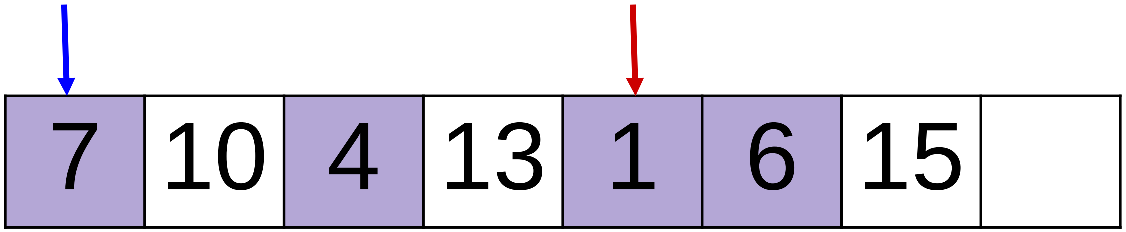
Let’s try known data structures:

- Array
- Sorted array
- Linked list

Array

- Range Search: $O(n)$ ✗

range(1,7)



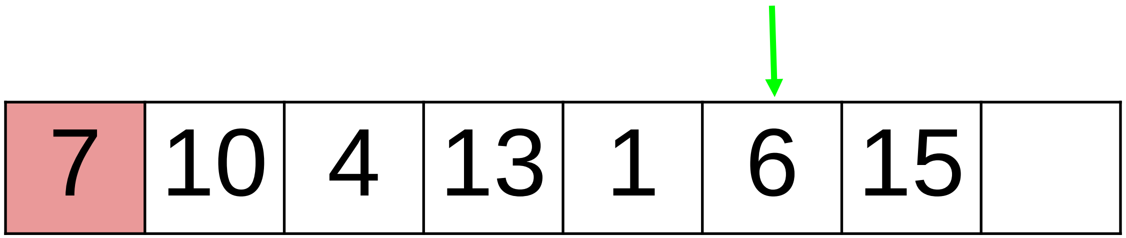
7	10	4	13	1	6	15	

Array

→ Range Search: $O(n)$ ×

→ Nearest Neighbor: $O(n)$ ×

nearestNeighbor(6)

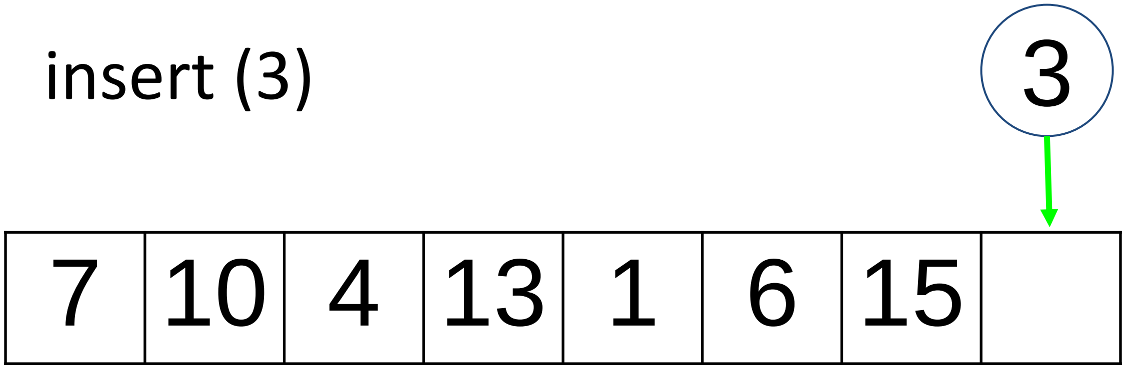


7	10	4	13	1	6	15	

Array

- → Range Search: $O(n)$ ×
- → Nearest Neighbor: $O(n)$ ×
- → Insert: $O(1)$ V

insert (3)

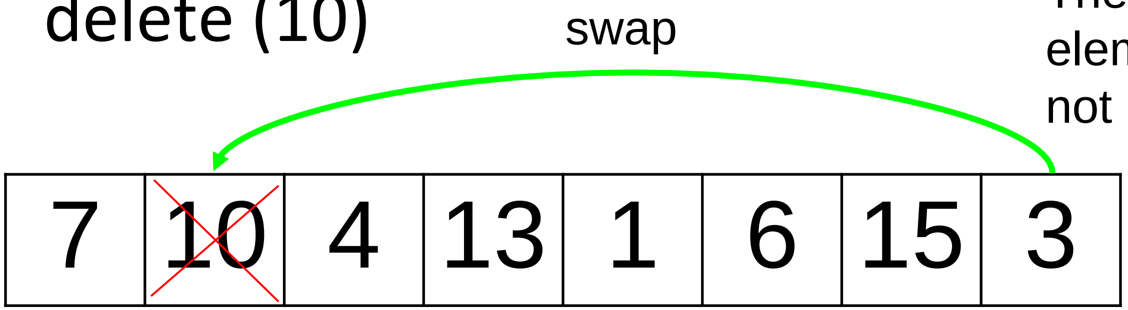


7	10	4	13	1	6	15	

Array

- Range Search: $O(n)$ ×
- Nearest Neighbor: $O(n)$ ×
- Insert: $O(1)$ V
- Remove: $O(1)^*$ V

delete (10)



swap

7	~~10~~	4	13	1	6	15	3

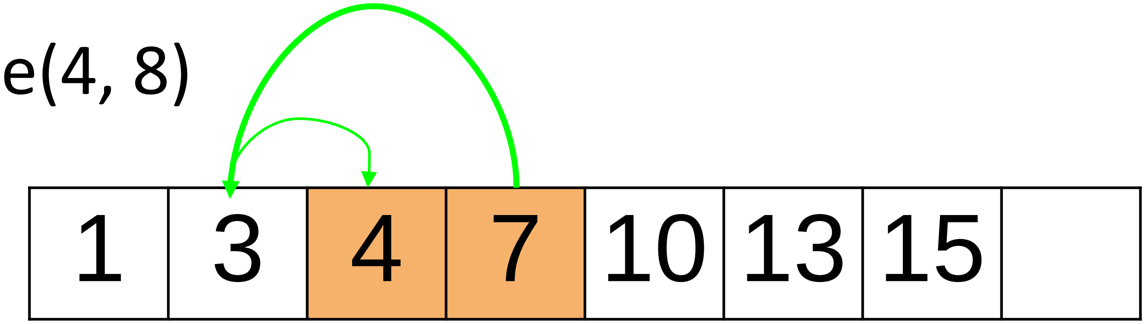
The order of
elements does
not matter!

After locating an index of the element to be removed

Sorted Array

→ Range Search: $O(\log(n))$ V

range(4, 8)

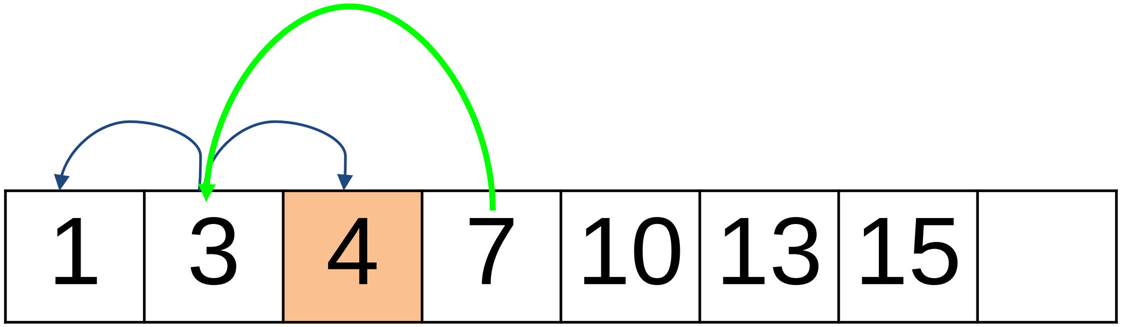


Sorted Array

➔ Range Search: $O(\log(n))$ V

➔ Nearest Neighbor: $O(\log(n))$ V

nearestNeighbor(3)

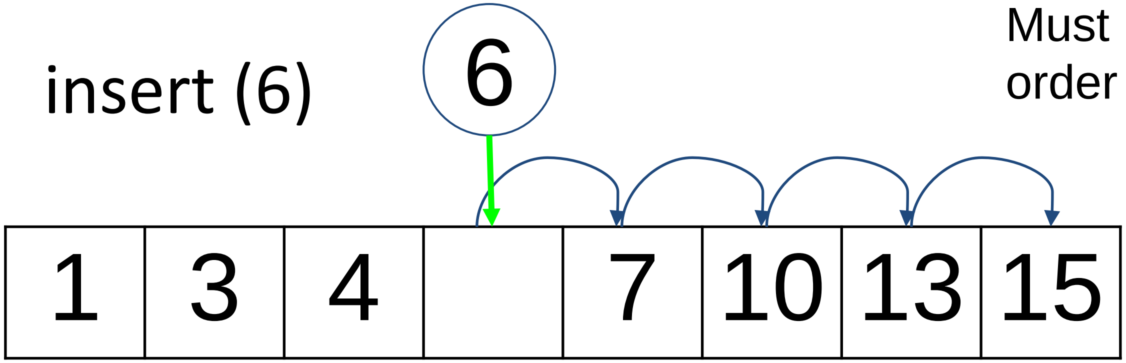


1	3	4	7	10	13	15	

Sorted Array

- ➔ Range Search: $O(\log(n))$ V
- ➔ Nearest Neighbor: $O(\log(n))$ V
- ➔ Insert: $O(n)$ ×

insert (6)



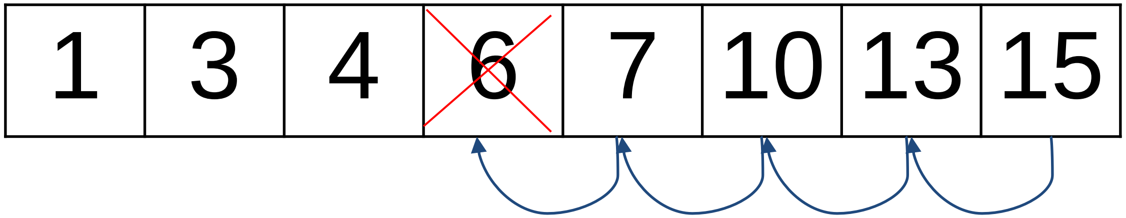
Must keep in sorted order – so shift

Sorted Array

- Range Search: $O(\log(n))$ V
- Nearest Neighbor: $O(\log(n))$ V
- Insert: $O(n)$ ×
- Remove: $O(n)$ ×

delete (6)

Cannot have gaps – shift again

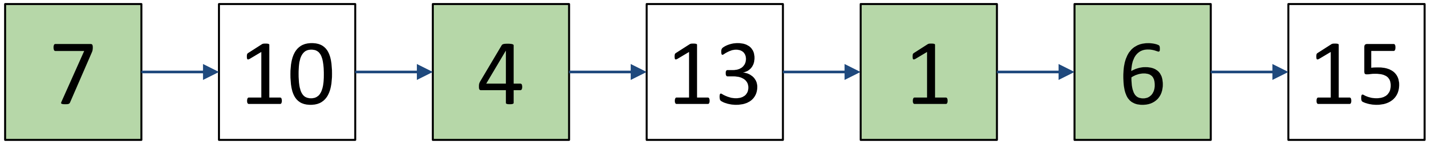


1	3	4	~~6~~	7	10	13	15

Linked List

- Range Search: $O(n)$ ×

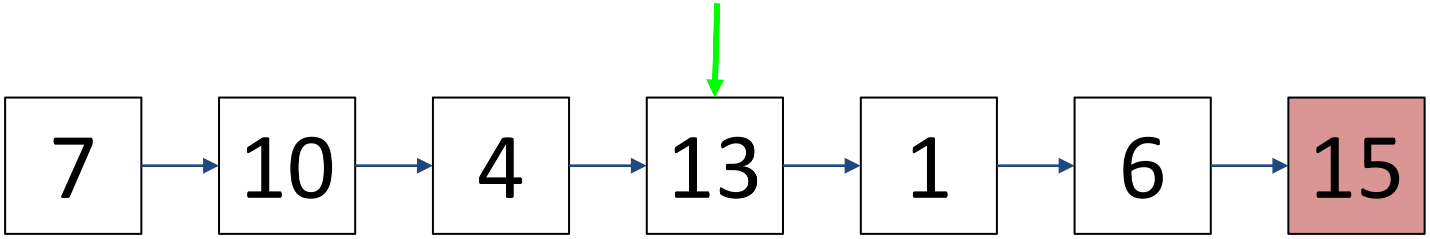
range (4, 9)



Linked List

- Range Search: $O(n)$ ×
- Nearest Neighbor: $O(n)$ ×

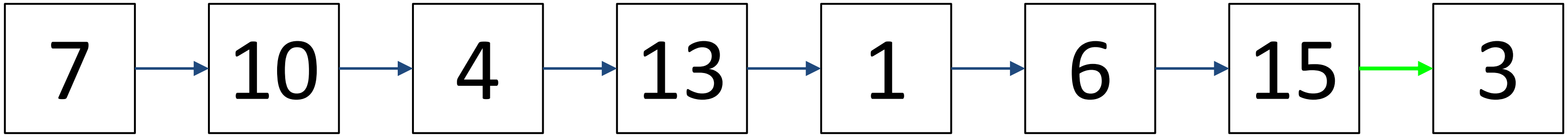
nearestNeighbor(13)



Linked List

- ➔ Range Search: $O(n)$ ×
- ➔ Nearest Neighbor: $O(n)$ ×
- ➔ Insert: $O(1)$ V

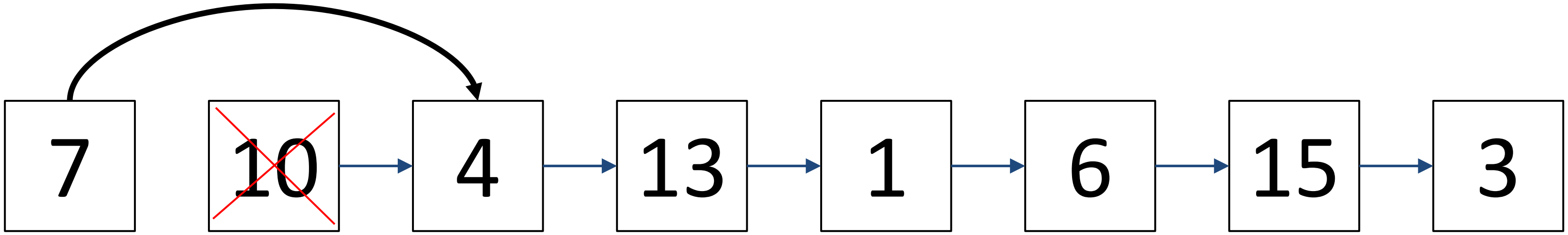
insert (3)



Linked List

→ Range Search:	$O(n)$	×
→ Nearest Neighbor:	$O(n)$	×
→ Insert:	$O(1)$	V
→ Remove:	$O(1)^*$	V

delete (10)



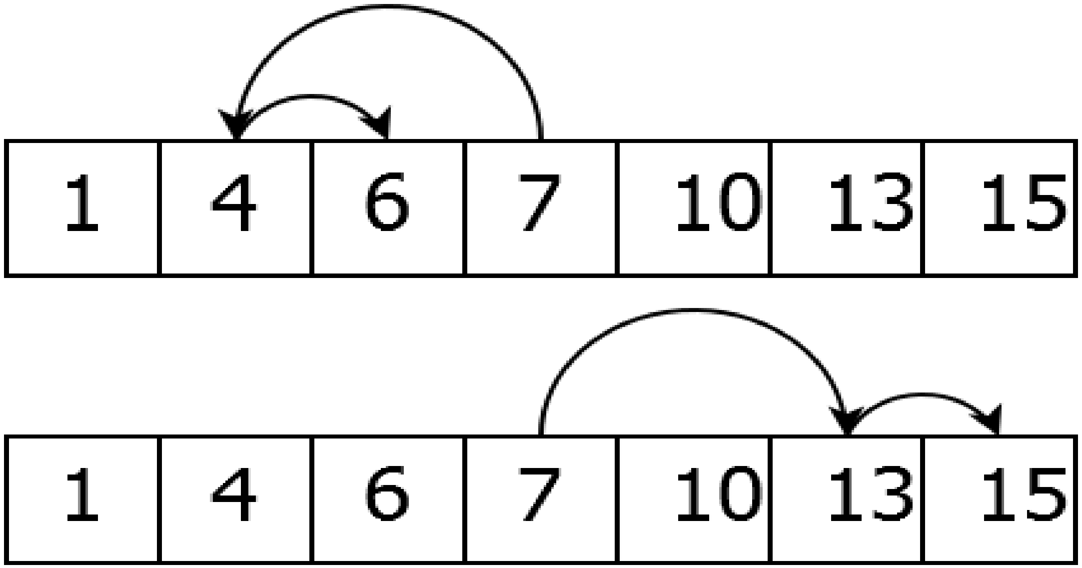
*after locating the node with the element to be removed

Nothing works!

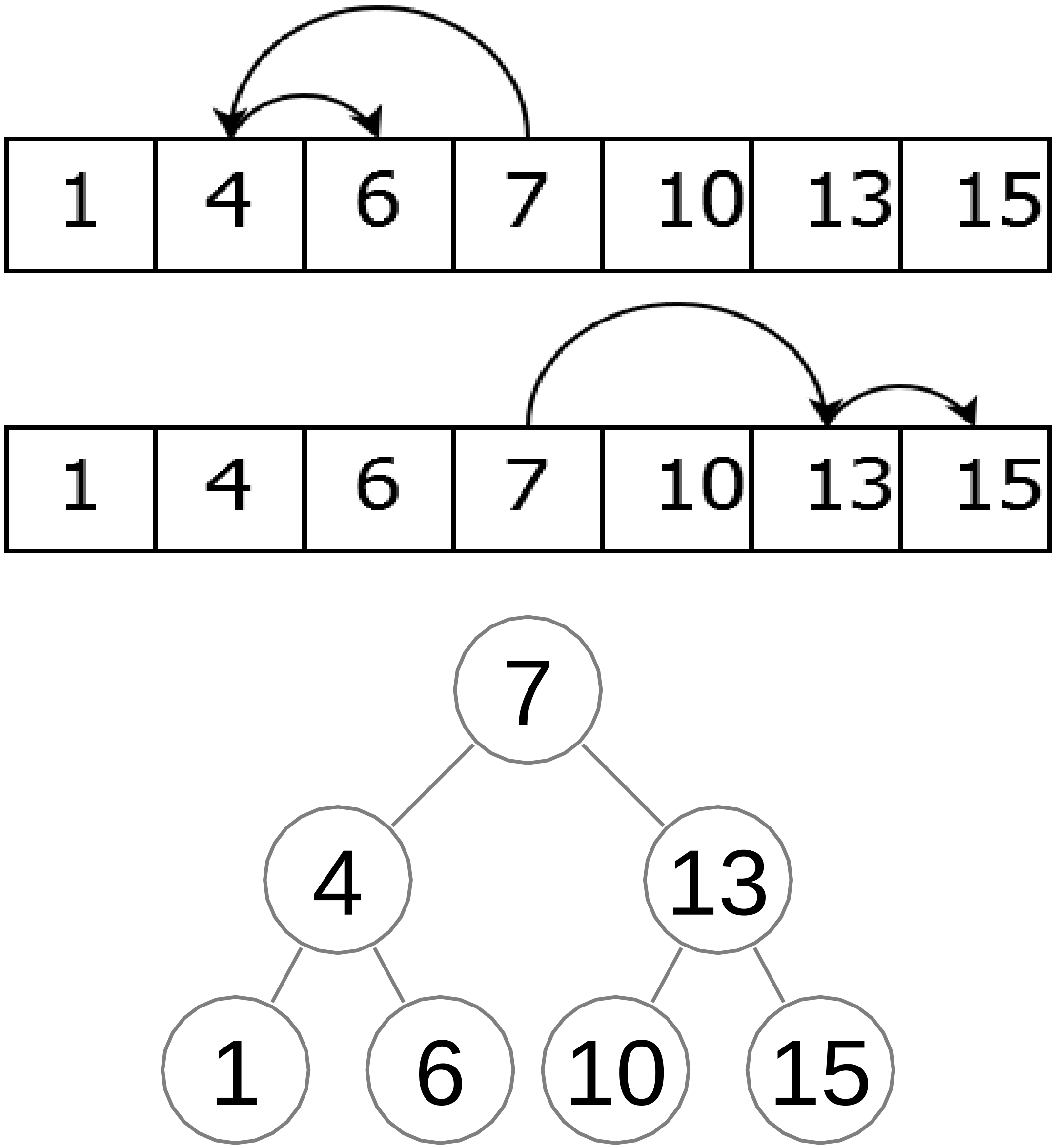
- We want an efficient data structure for fast search **and** update operations
- None of the known data structures work
- Sorted arrays are good for search but not for update

We need something new…

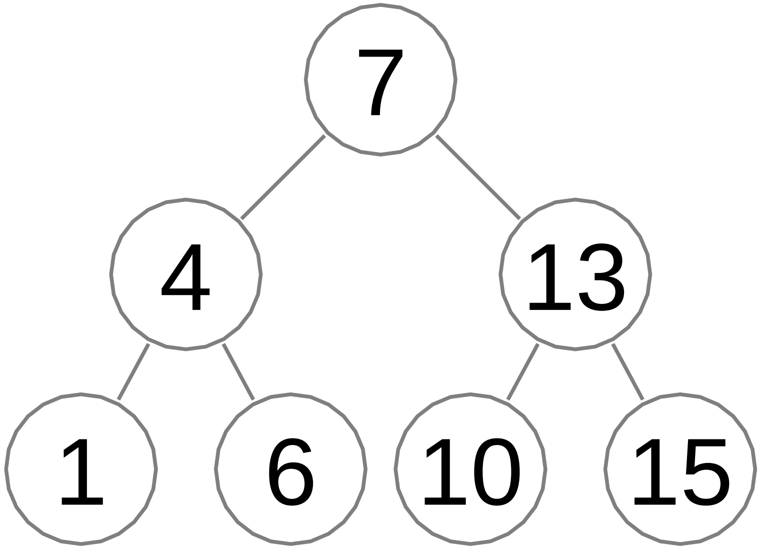
Recall: Binary Search



What if we record search questions…



We will get a tree

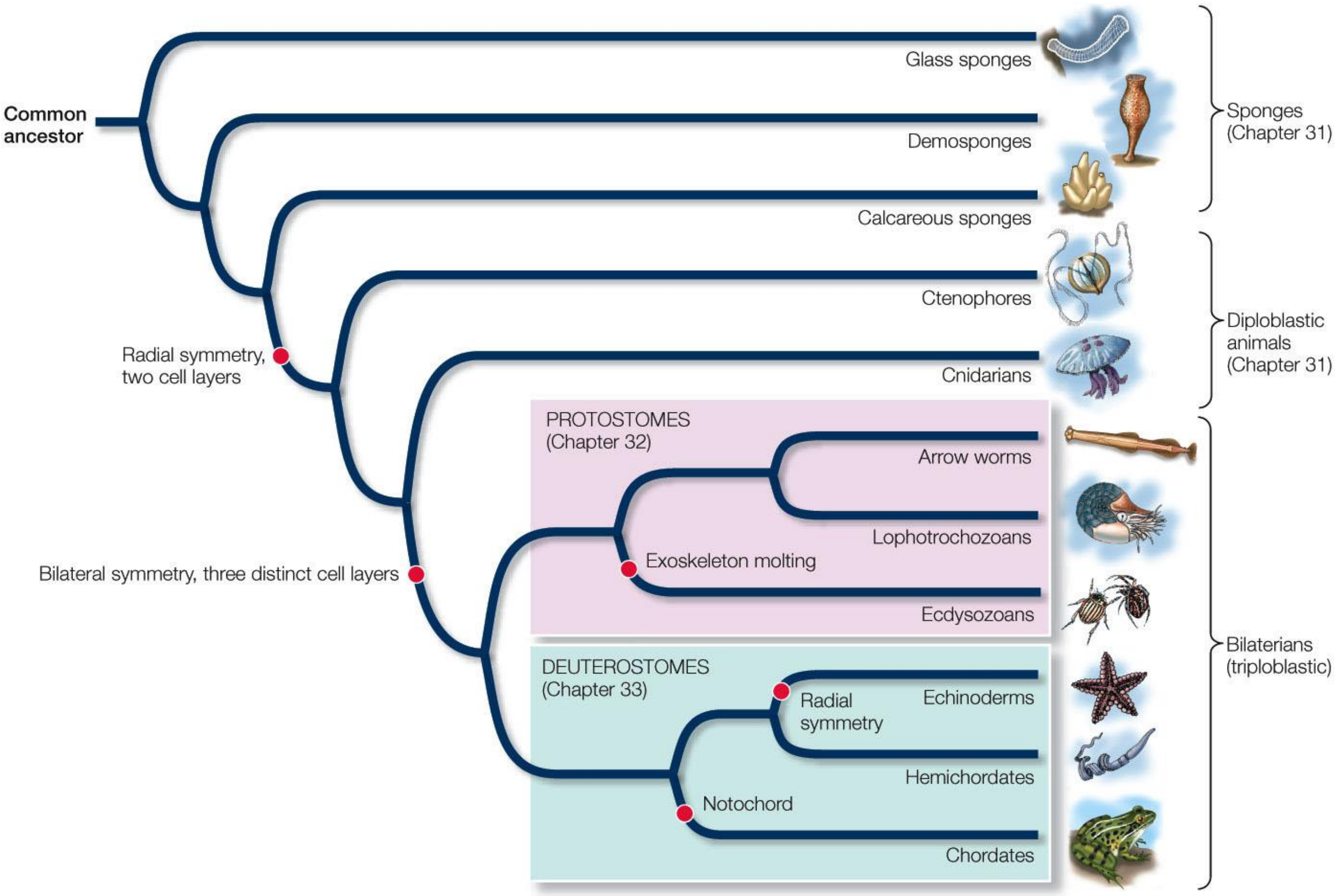


Binary Search Tree

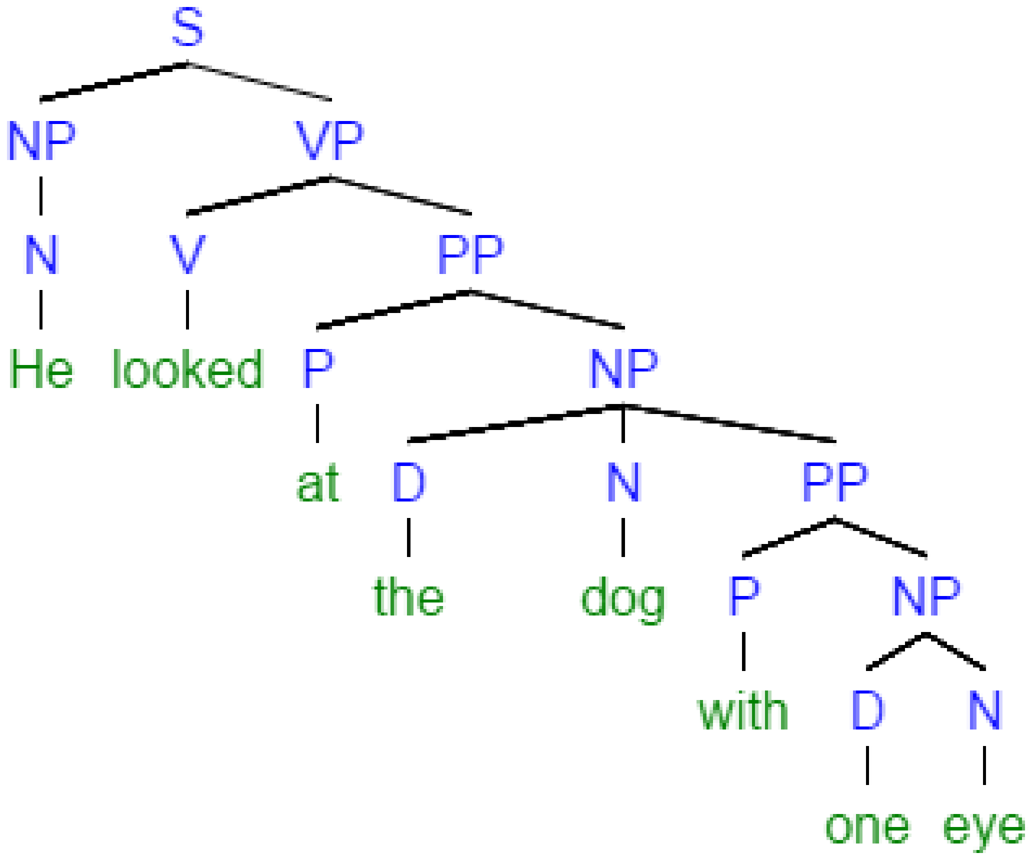
New Data Structure: *Tree*

Natalie Jeremijenko, Tree Logic,
Massachusetts Museum of Contemporary Art
(MASS MoCA), 1999

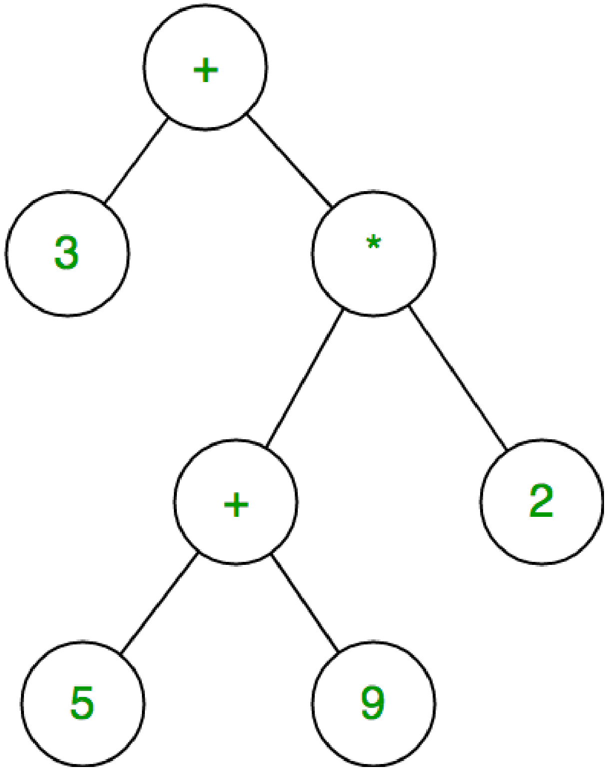
Biology:
Phylogenetic Tree of animals



Natural Language Processing: Syntax Tree

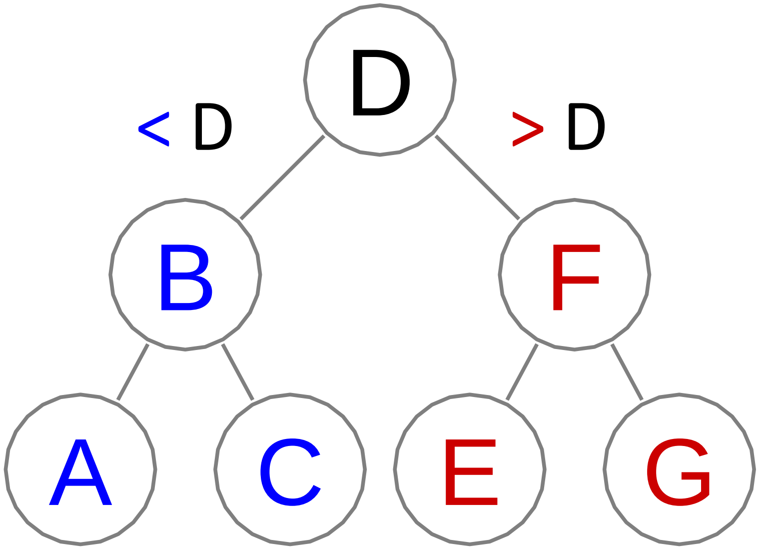


Computer programs: Expression Tree



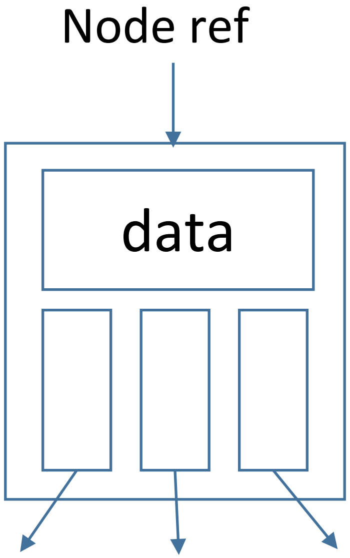
$3 + ((5 + 9) * 2)$

Quick Search:
Binary Search Tree



Tree - new recursive data structure

- Main element of the tree: *node*
- Each node contains data and an **array** of links to the child nodes



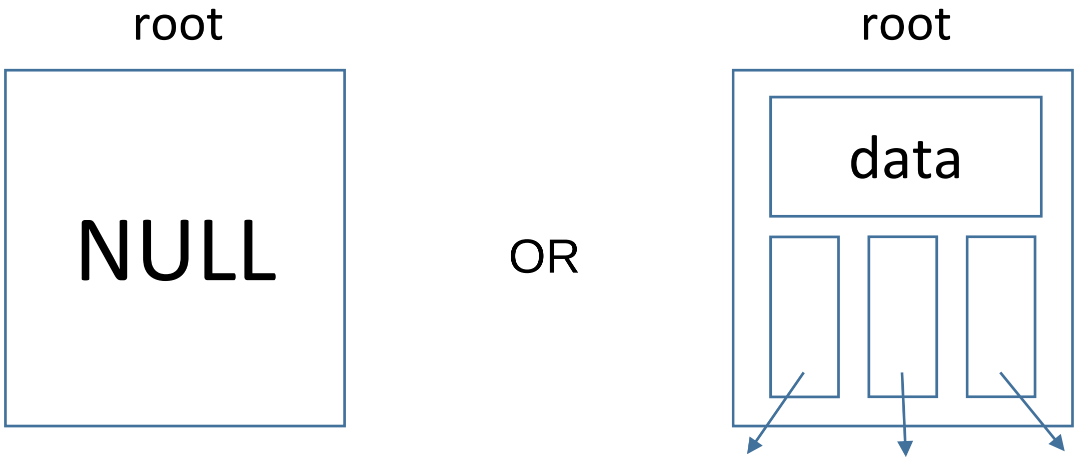
```
class TreeNode {
    int data;
    TreeNode [] children;
    [TreeNode parent;]

}
```

```
class TreeNode:
    def __init__(self, data):
        self.data = data
        self.children = []
        [self.parent = None]
```

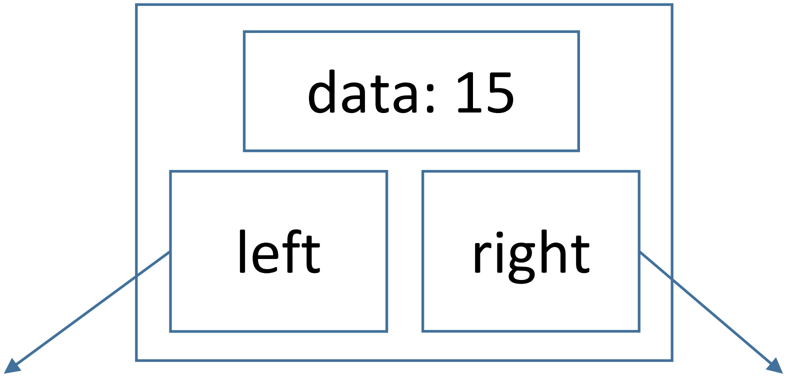
Tree is defined by a single reference variable ***root***

Tree is either

- Null (empty tree)
- Root node which contains data and links to child nodes



Binary tree:
each node has 2 children



data: 15

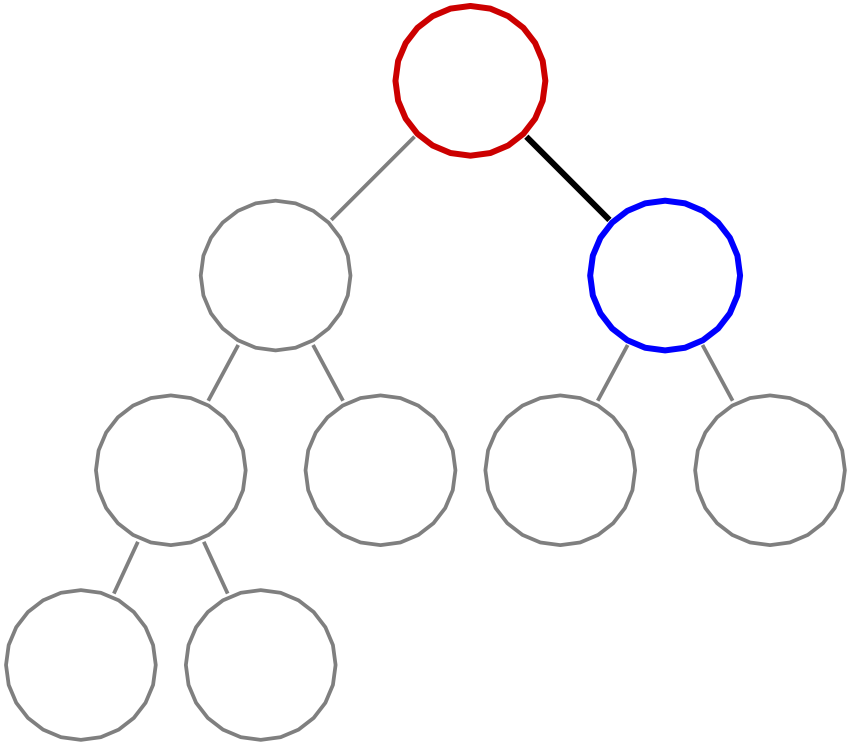
left

right

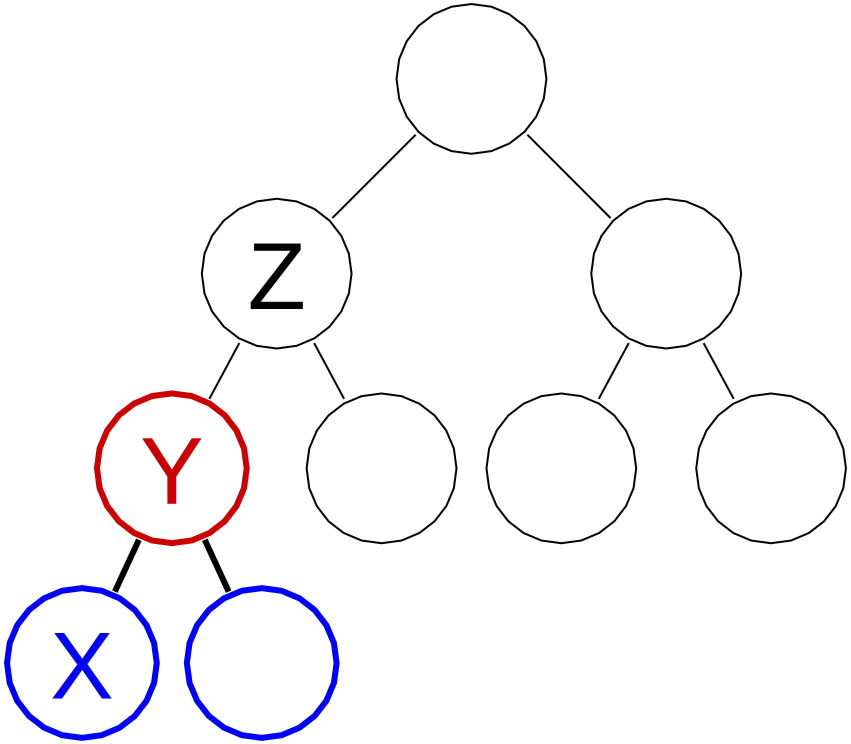
Either Left or Right can be null (empty tree)

```
class Node {
     int data;
     Node left;
     Node right;
     [Node parent;]

}
```

Tree terminology: parent and child

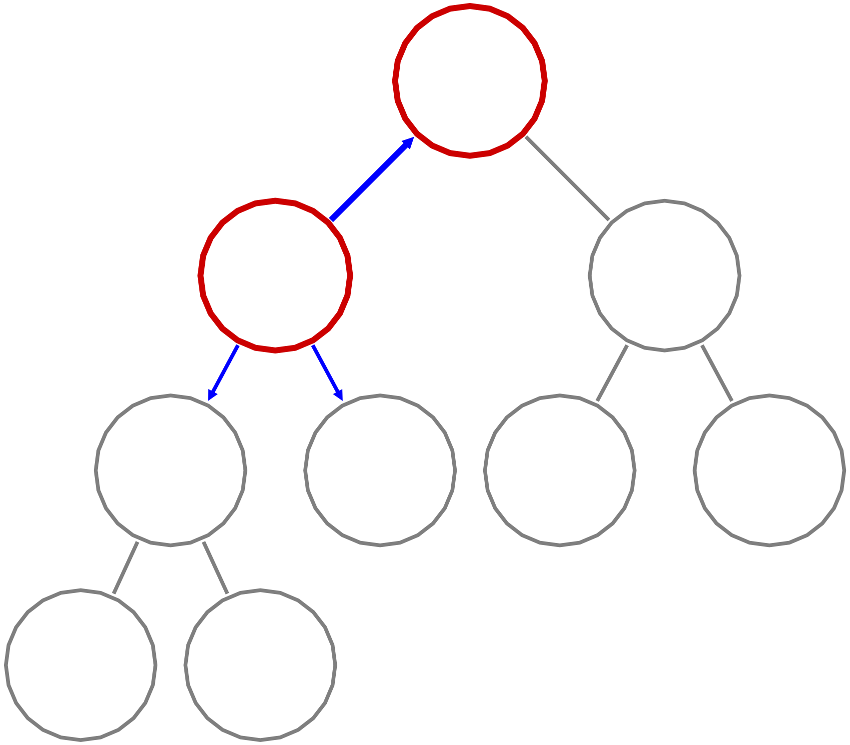


Tree terminology: parent and child



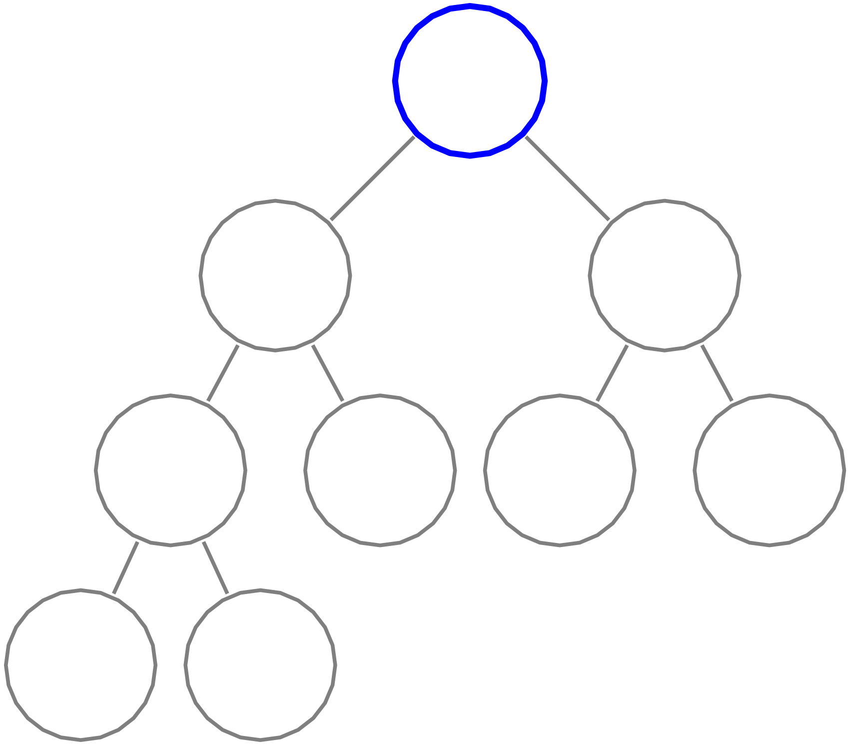
Have direct relationship

Tree terminology: node and edge



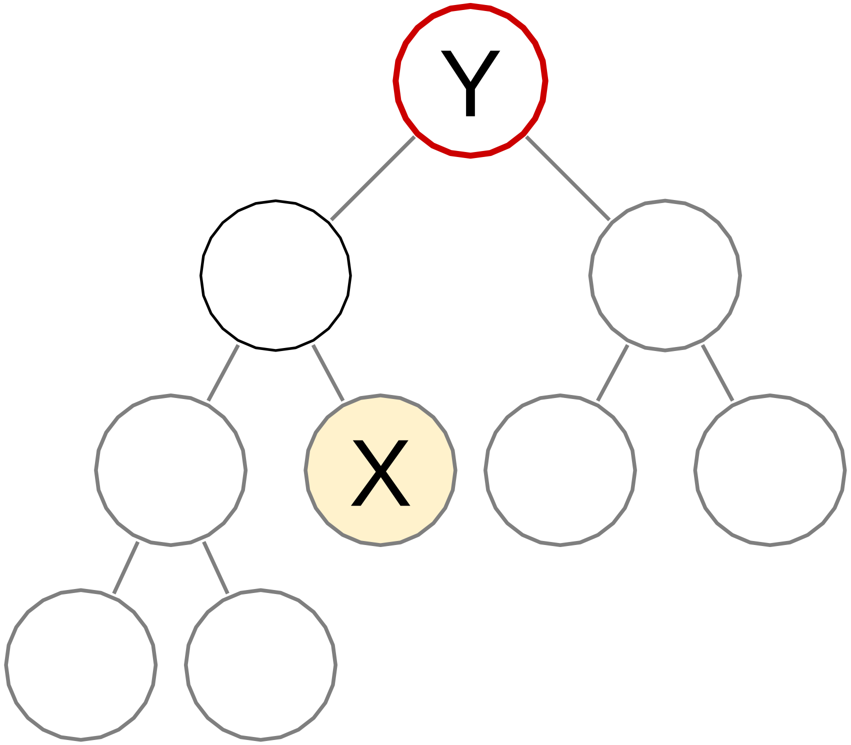
An edge connects nodes:
parent-child or child-parent relationships

Tree terminology: root



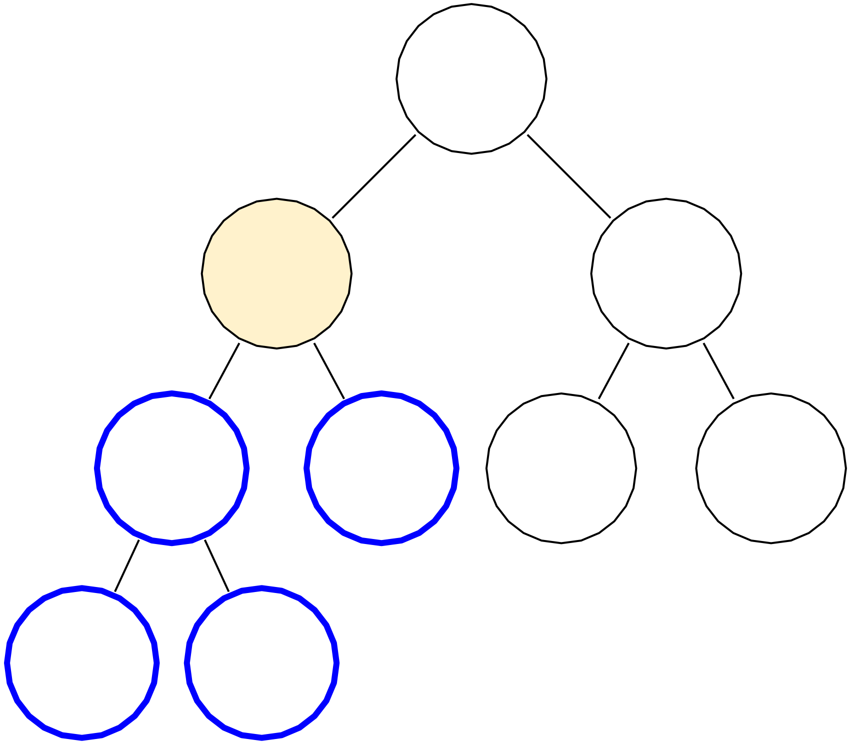
The parent of all nodes, the starting point

Tree terminology: ancestor and descendant



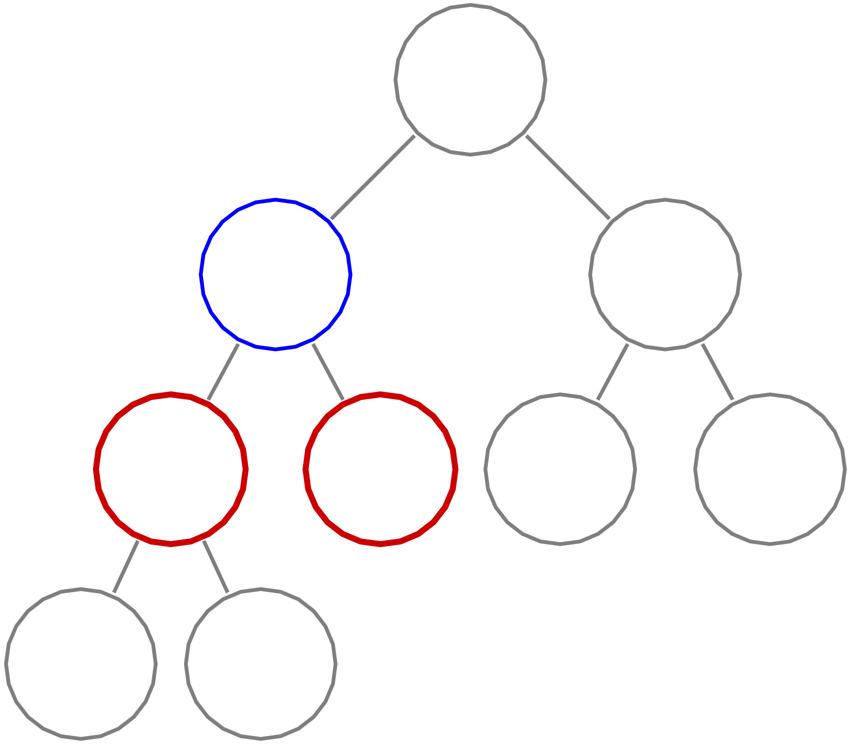
Ancestor: parent, or parent of parent, etc.

Tree terminology: ancestor and descendant



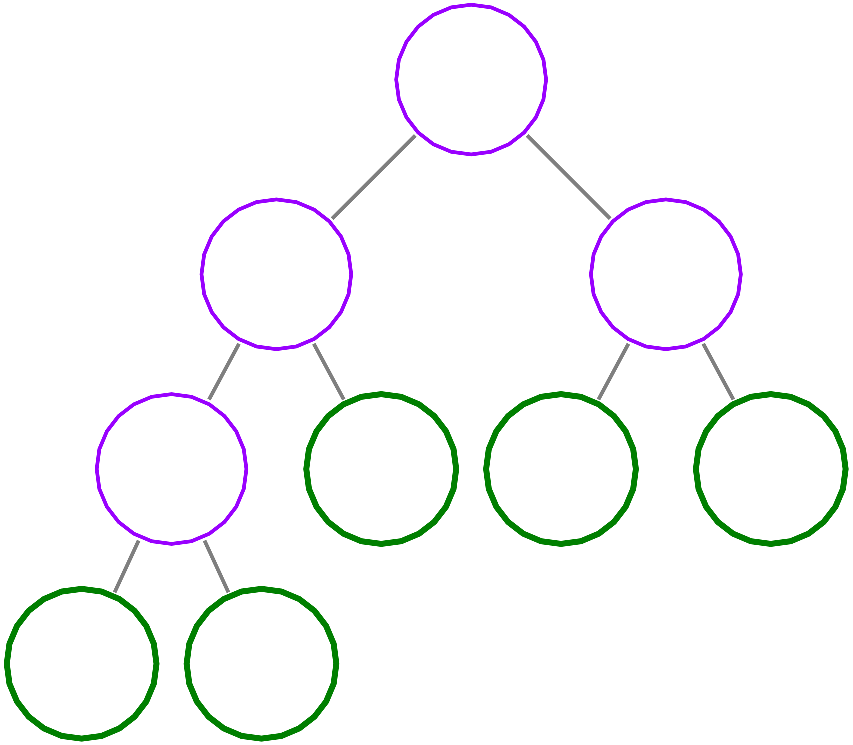
Descendant: child, or child of child, etc.

Tree terminology: siblings



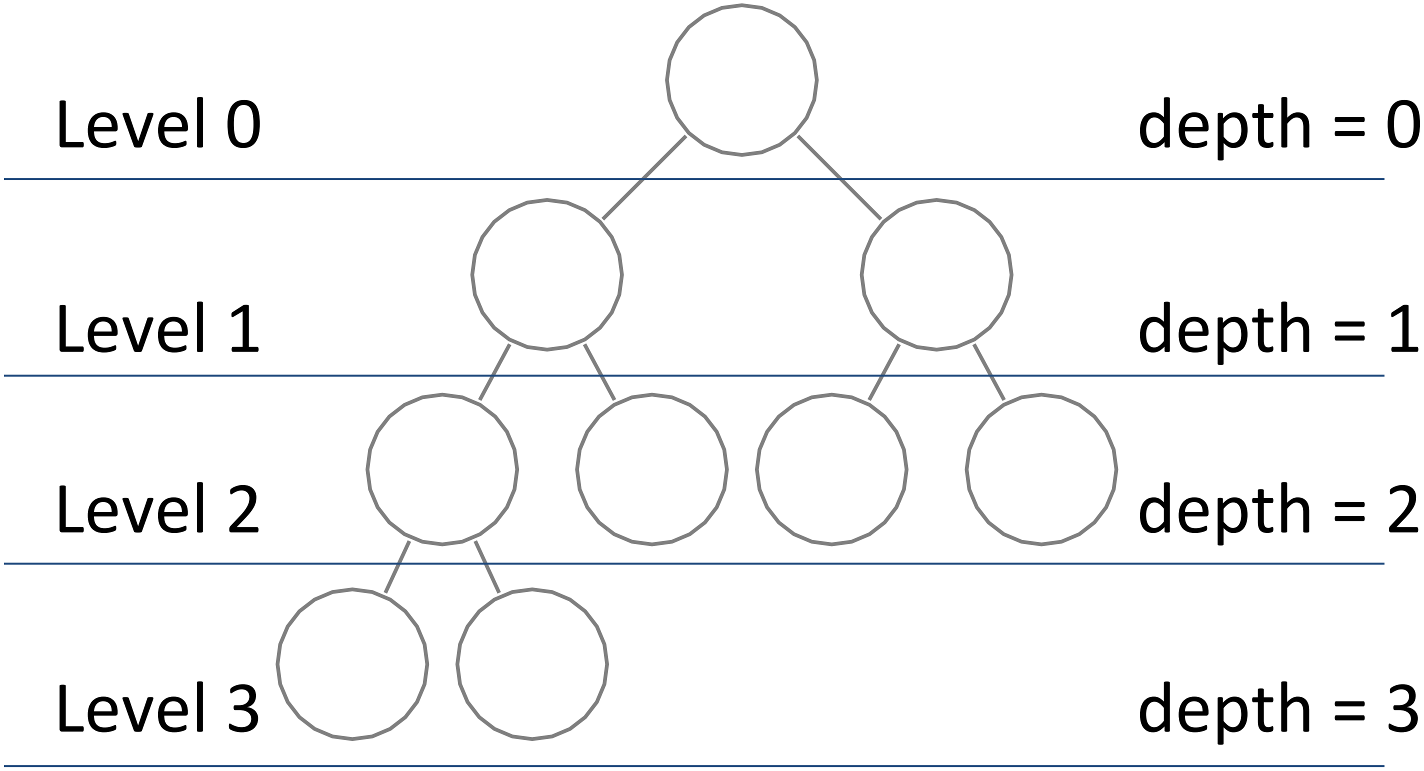
Sharing the same parent

Tree terminology: leaves and interior (internal) nodes



In a leaf node both children are empty trees

Tree levels and node depth



Level 0

depth = 0

Level 1

depth = 1

Level 2

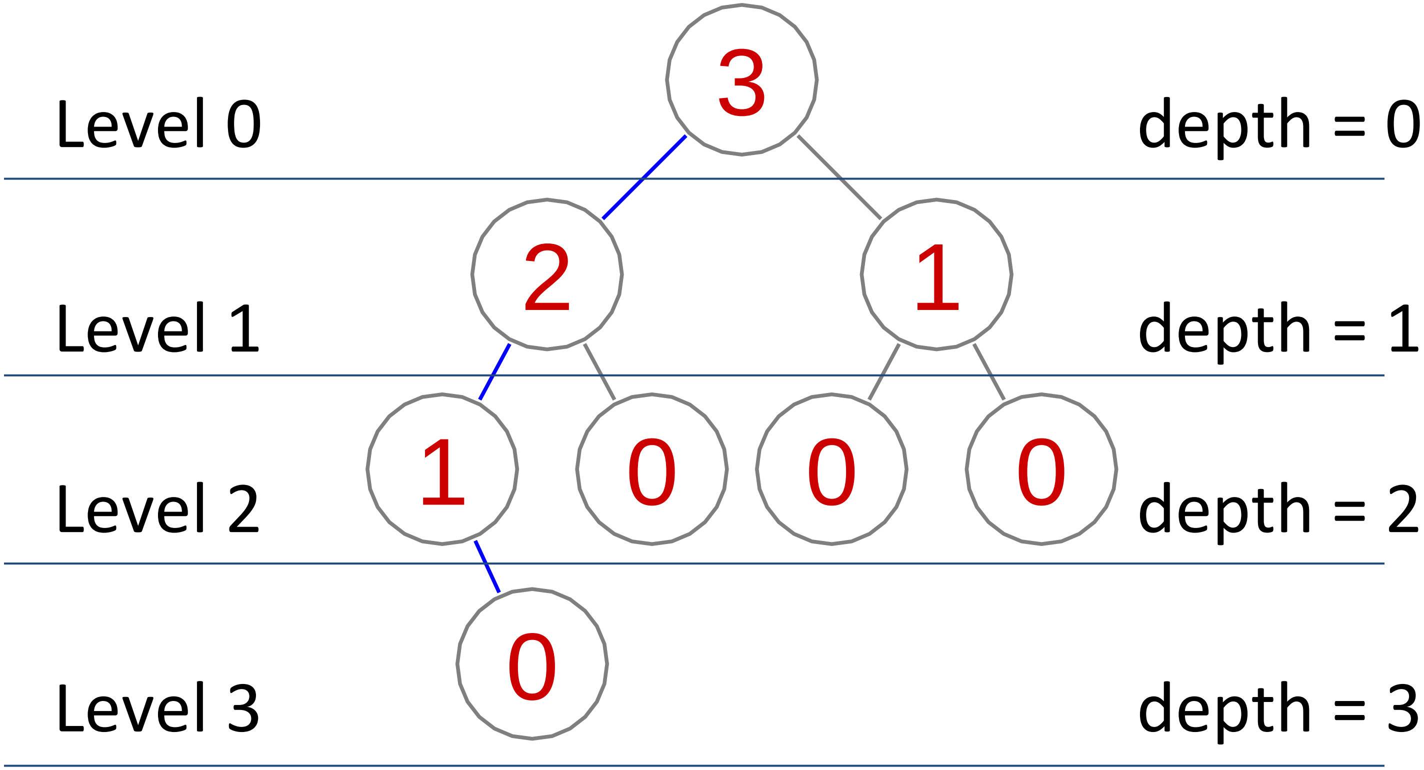
depth = 2

Level 3

depth = 3

Distance from the root:
how many edges to go from the root to the node

Node height



Level 0 depth = 0

Level 1 depth = 1

Level 2 depth = 2

Level 3 depth = 3

Distance from the node to the bottom:
how many edges to go to the furthest leaf

Algorithm *height* (*node*)

```
if node == null :
    return 0
if node.left == null and node.right == null:
    return 0
return 1 + Max(height(node.left),
               height(node.right))
```

Algorithm *size* (*tree*)

```
if tree == null
   return 0
return 1 + size(tree.left) + size(tree.right)
```

Recursive algorithms are common

Which of the following correctly computes the *depth* of a given tree node in the non-empty tree?

A.

```
Algorithm depth(node)
    if $node == null$
        return 0
    return $1 + depth(node.parent)$
```

B.

```
Algorithm depth(node)
    if $node.parent == null$
        return 0
    return $1 + depth(node.parent)$
```

C.

```
Algorithm depth(node)
    if $node == null$
        return -1
    return $1 + depth(node.parent)$
```

D. More than one is correct

E. None is correct

Tree traversals

- Task: list all the nodes in the tree

Two types of traversals:

- *Depth-first*: we completely traverse one sub-tree before exploring a sibling sub-tree
- *Breadth-first*: We traverse all nodes at one level before progressing to the next level

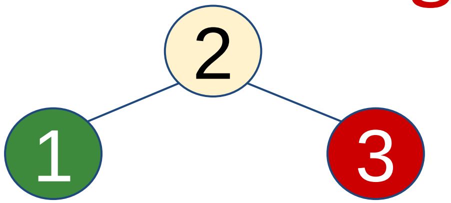
Depth-first tree traversals

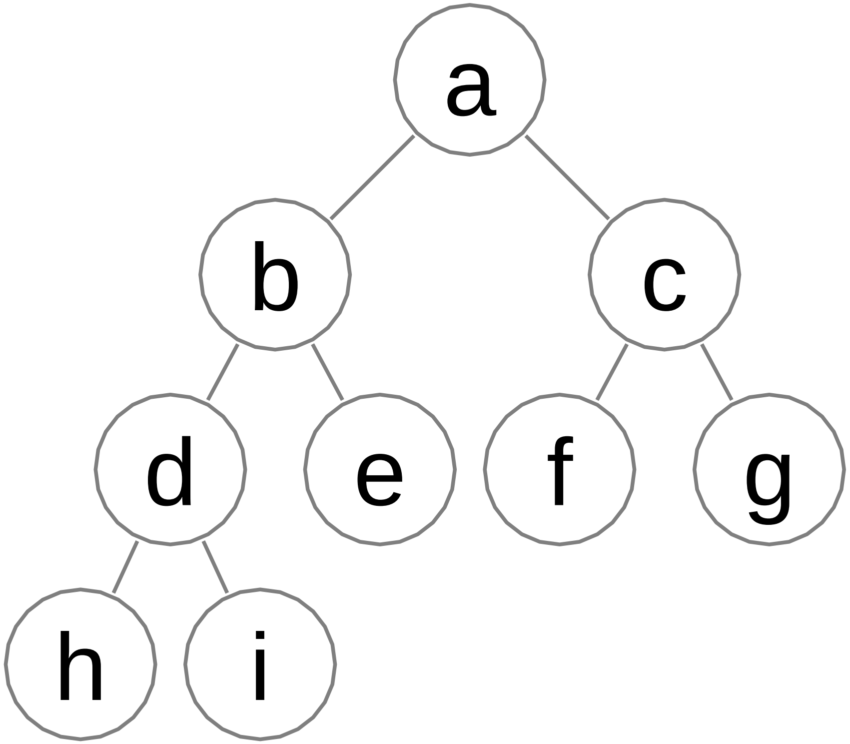
- In-order
- Pre-order
- Post-order

Depth-first: in order

```
Algorithm InOrderTraversal(tree)
if tree == Null :
    return
InOrderTraversal(tree.left)
print (tree.key )
InOrderTraversal(tree.right)
```

left - me - right

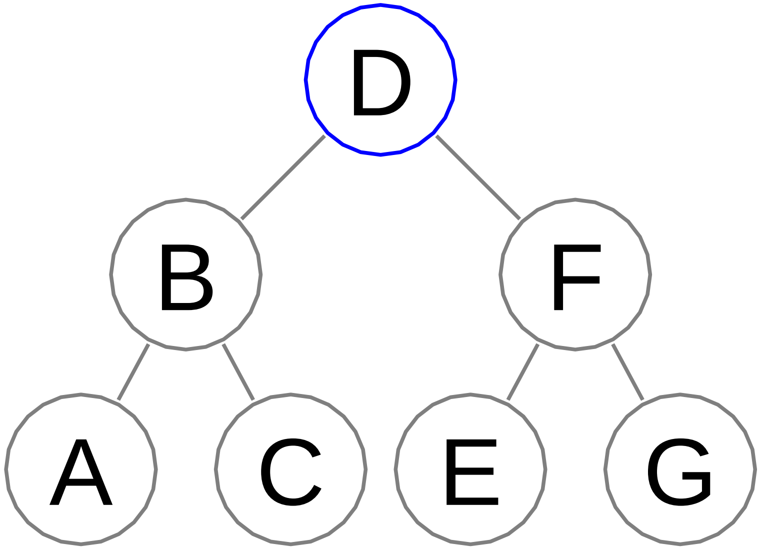




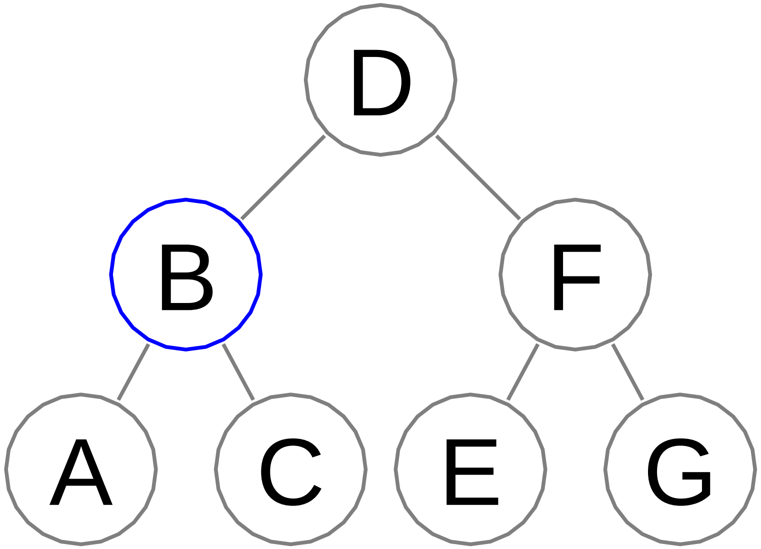
Which sequence of nodes is obtained as a result of in-order traversal of the tree on the left?

A. abdhiecfg
B. hdibeafcg
C. ahdibefcg
D. More than one is correct
E. None is correct

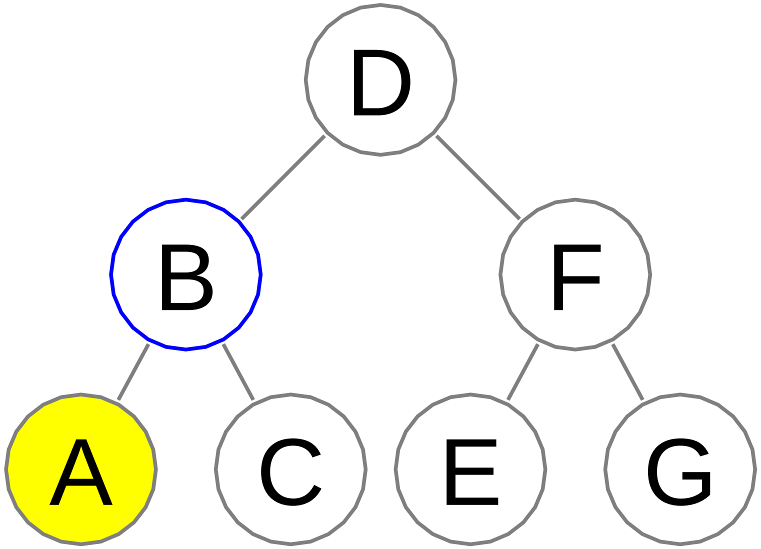
In-order



In-order

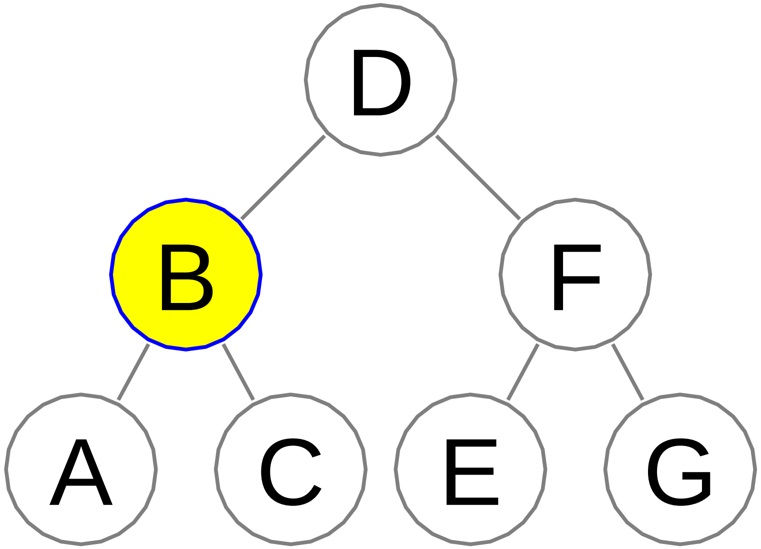


In-order



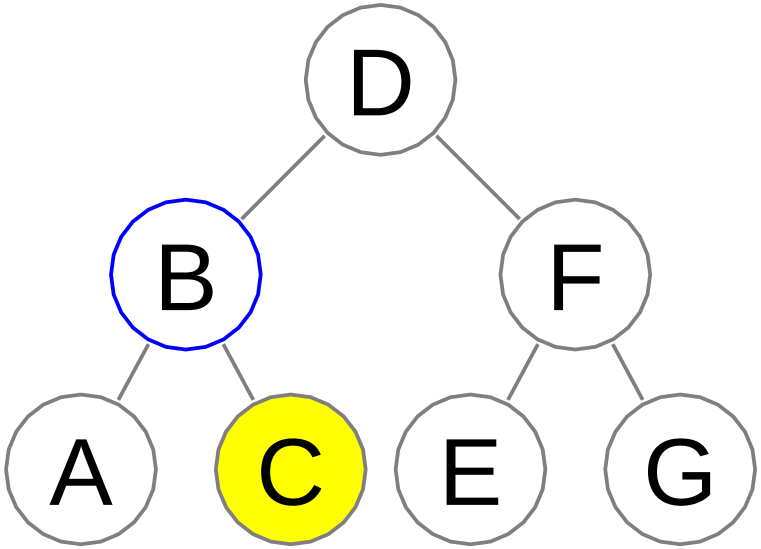
A

In-order



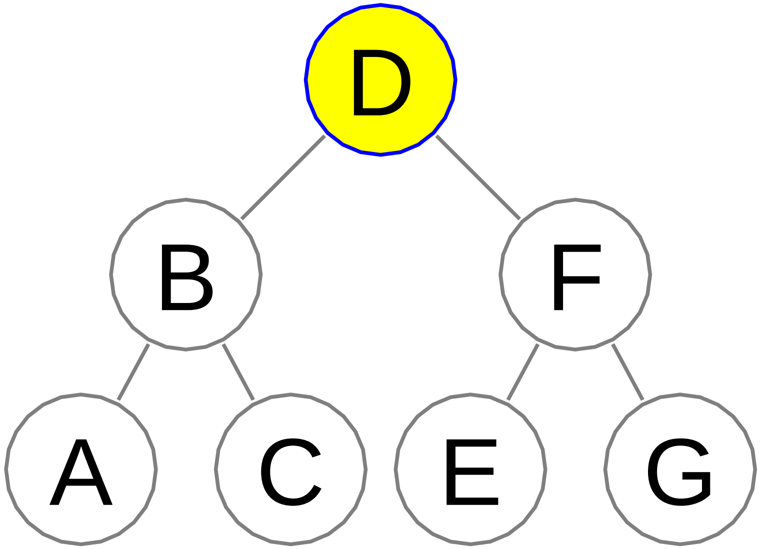
A B

In-order



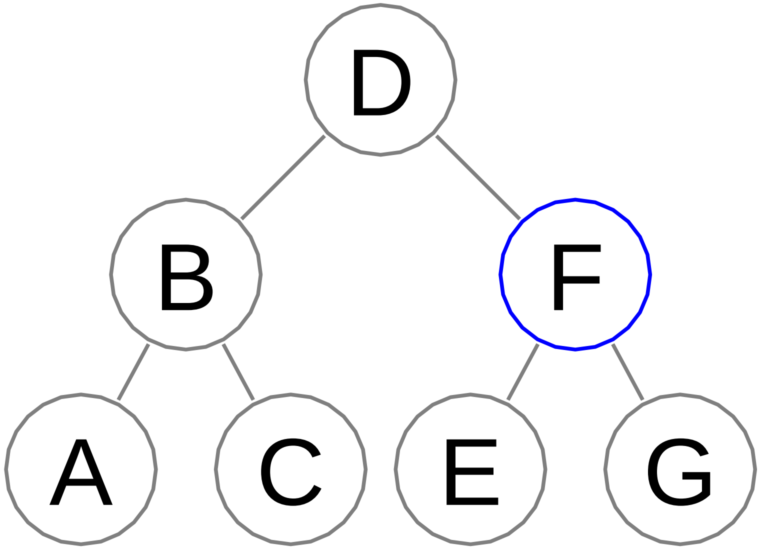
A B C

In-order



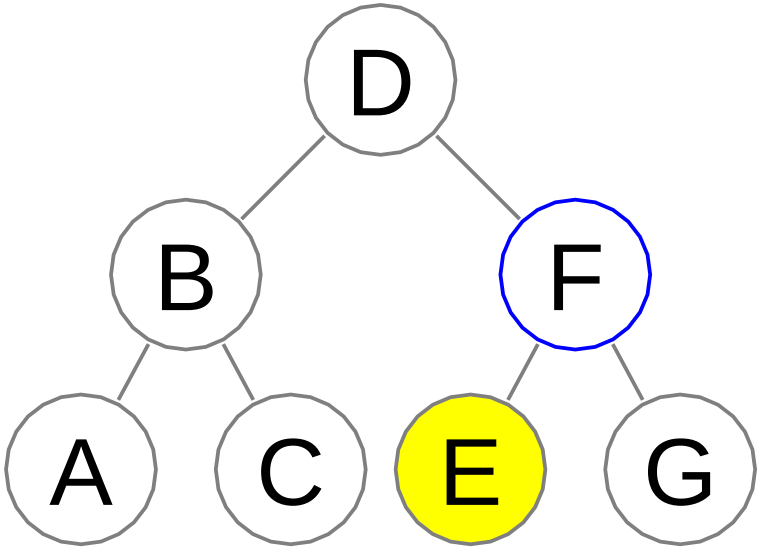
A B C D

In-order



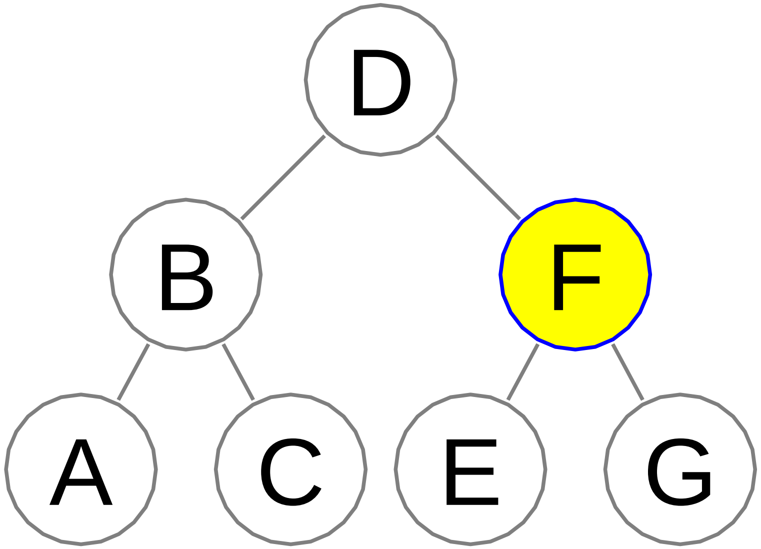
A B C D

In-order



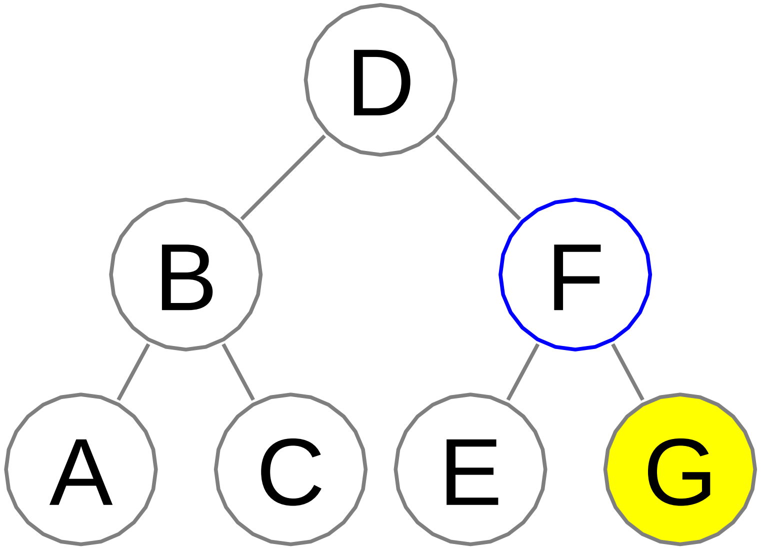
A B C D E

In-order



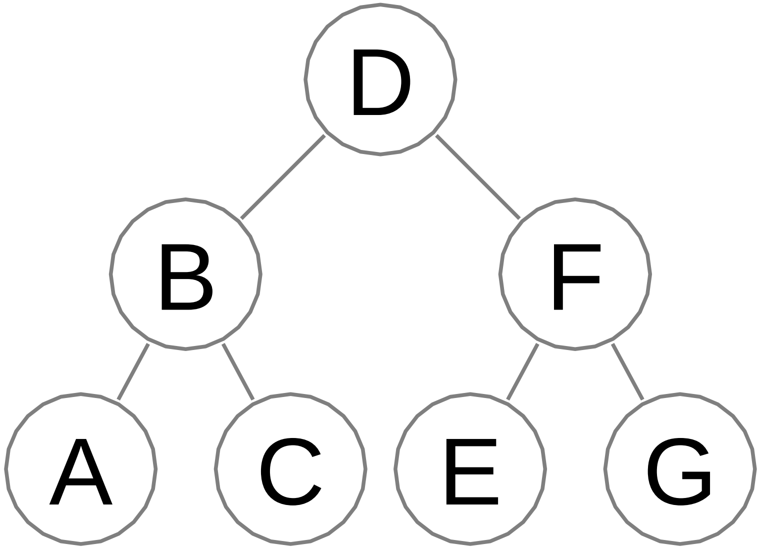
A B C D E F

In-order



A B C D E F G

In-order



me,
node D

A B C D E F G

left subtree of D

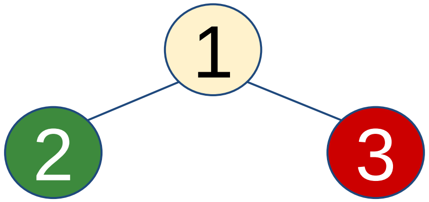
right subtree of D

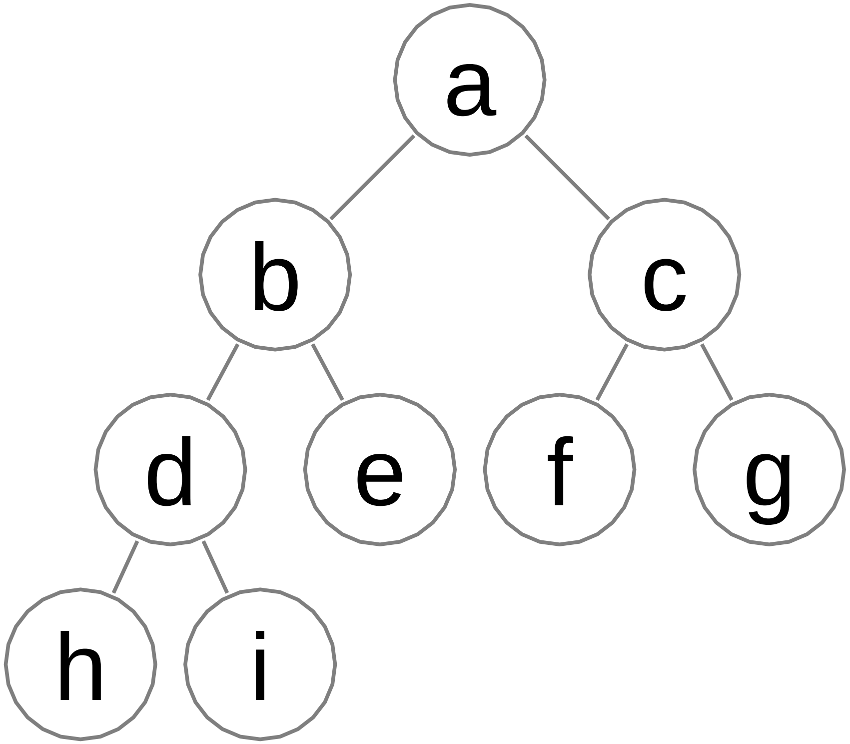
Depth-first: pre-order

```
Algorithm PreOrderTraversal(tree)
if  tree == null:
    return
print (tree.key)
PreOrderTraversal(tree.left)
PreOrderTraversal(tree.right)
```

me first

me → left → right

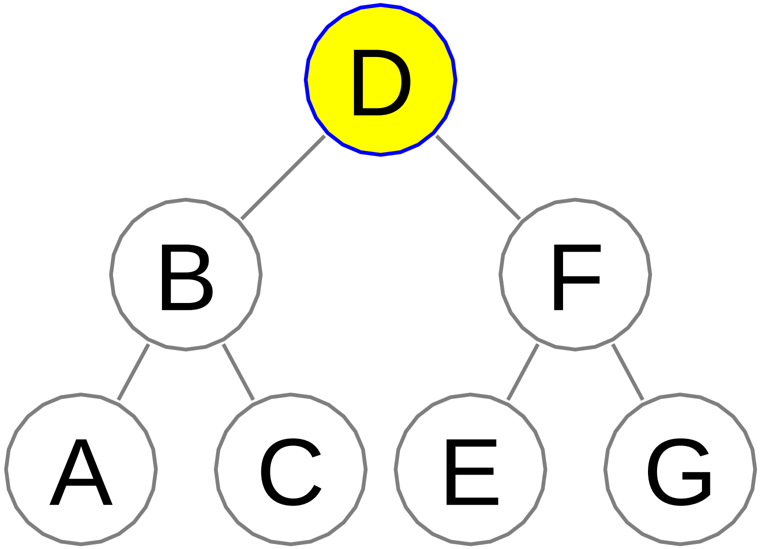




Which sequence of nodes is obtained as a result of pre-order traversal of the tree on the left?

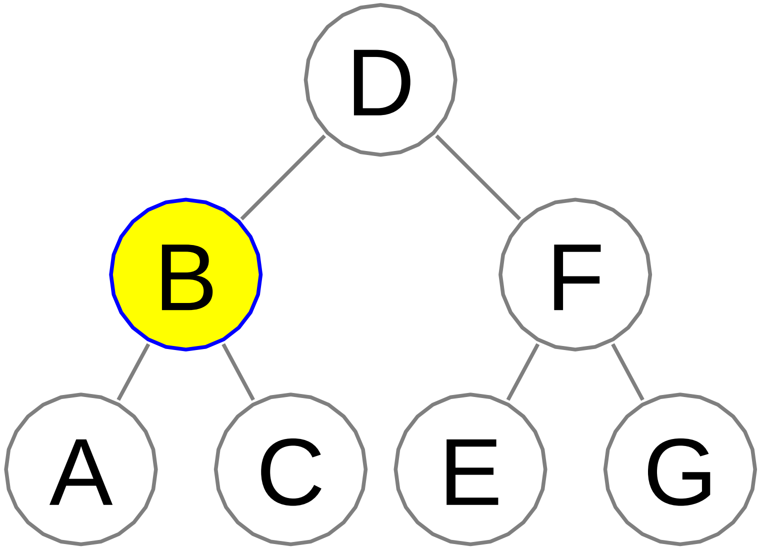
A. abdhiecfg
B. abcdehifg
C. abdhiecfg
D. More than one is correct
E. None is correct

Pre-order



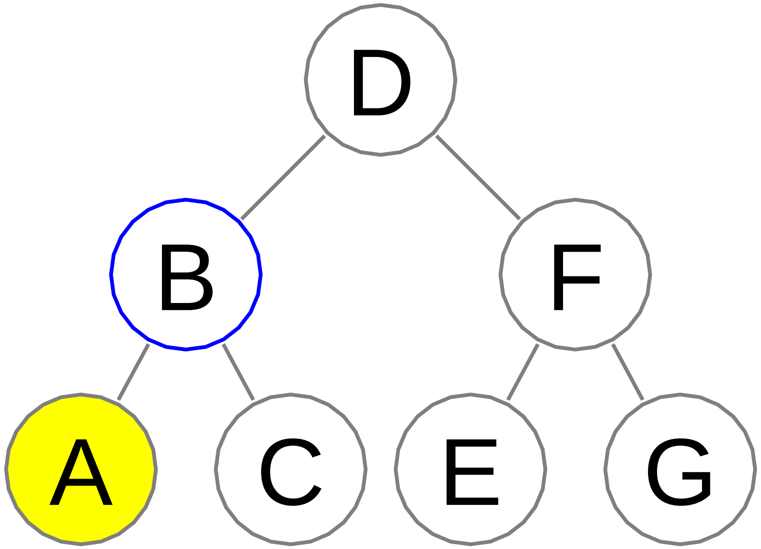
D

Pre-order



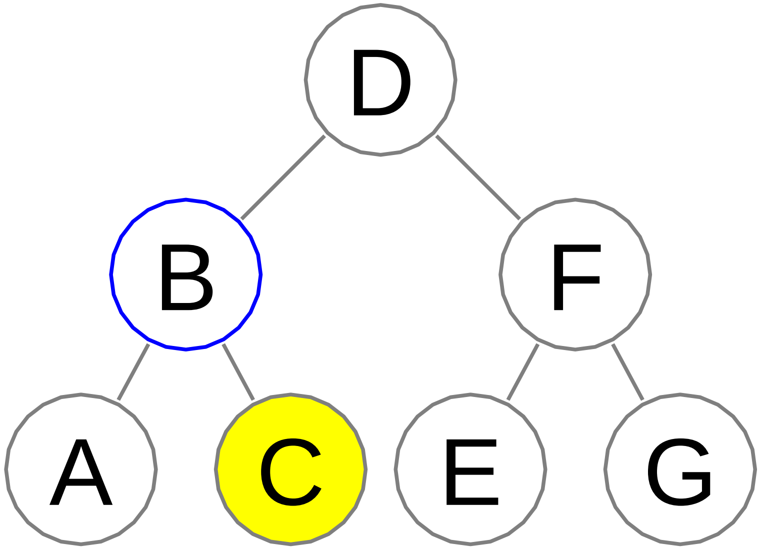
D B

Pre-order



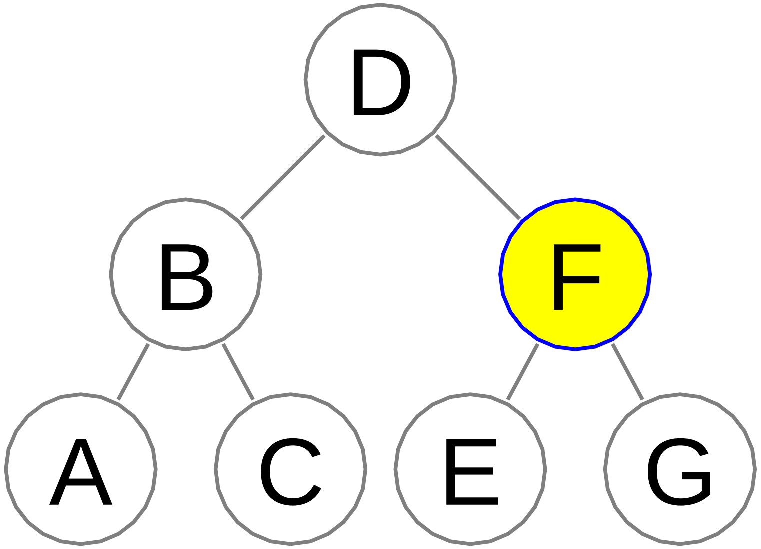
D B A

Pre-order



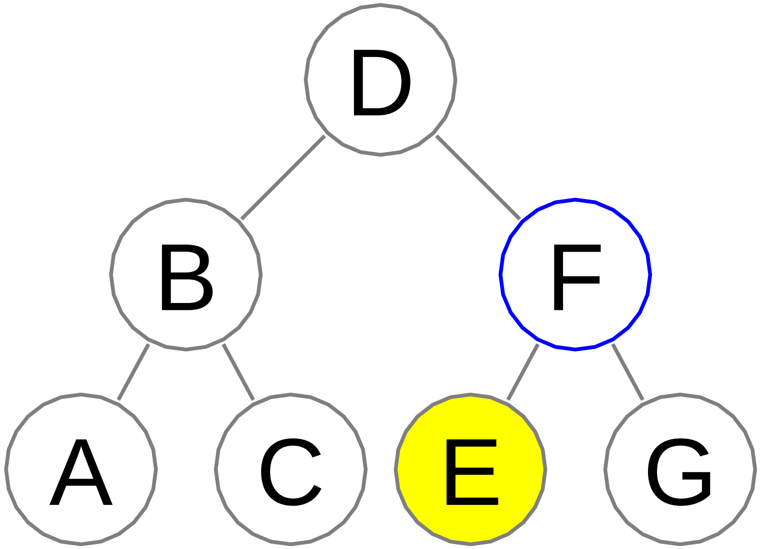
D B A C

Pre-order



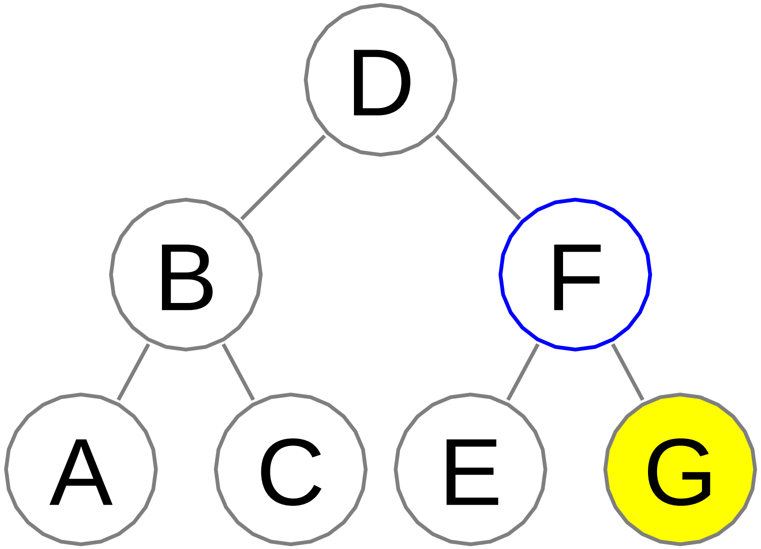
D B A C F

Pre-order



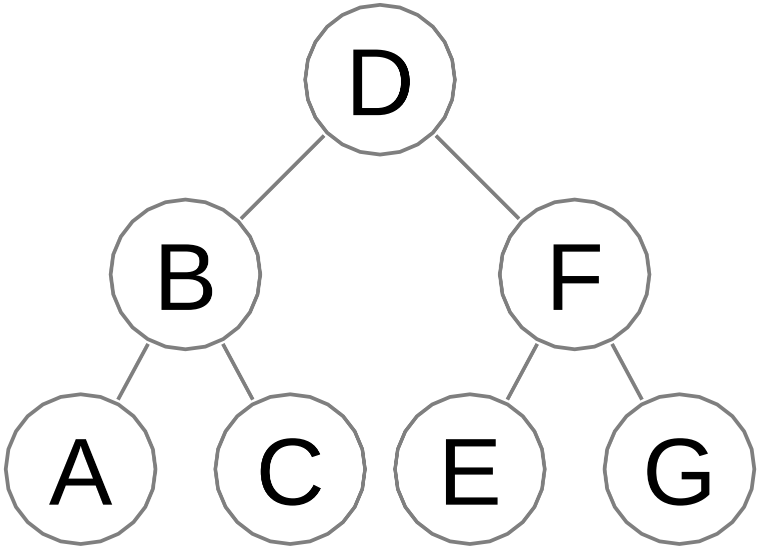
D B A C F E

Pre-order



D B A C F E G

Pre-order



me,
node D

D B A C F E G

left subtree of D right subtree of D

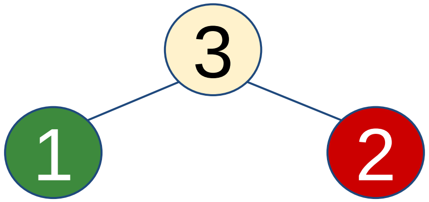
Depth-first: post-order

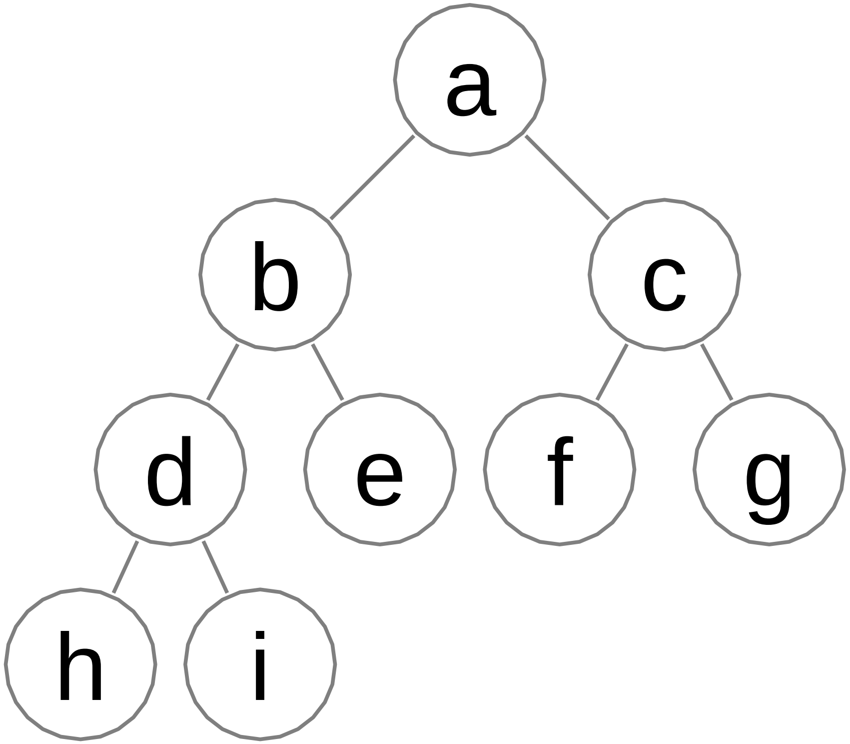
Algorithm *PostOrderTraversal*(*tree*)

```
if tree == null:
    return
PostOrderTraversal(tree.left)
PostOrderTraversal(tree.right)
print(tree.key)
```

children first

left → right → me

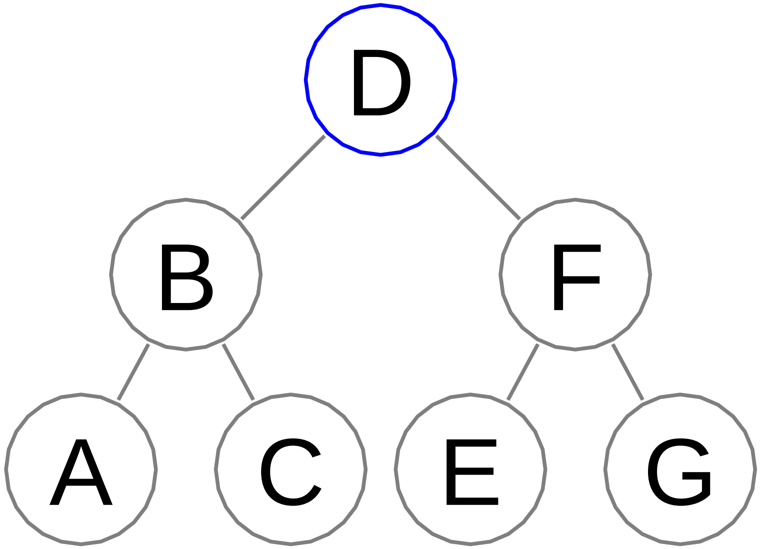




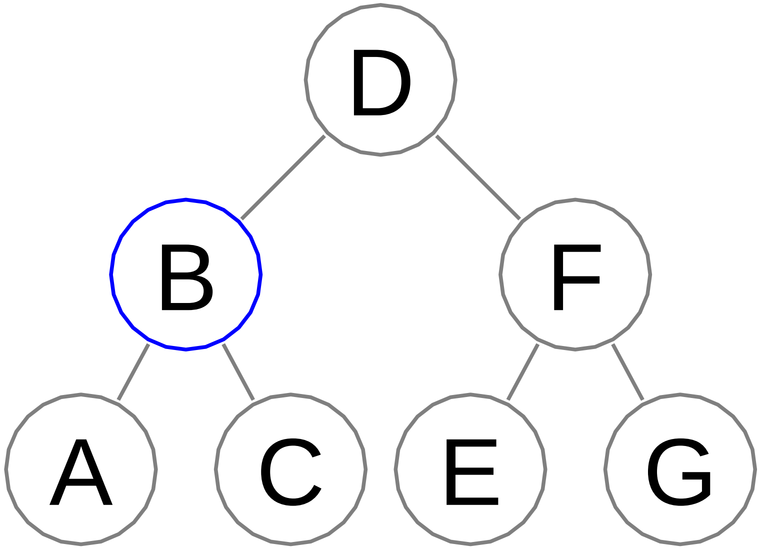
Which sequence of nodes is obtained as a result of post-order traversal of the tree on the left?

A. abdhiecfg
B. abcdehifg
C. hidebfgca
D. More than one is correct
E. None is correct

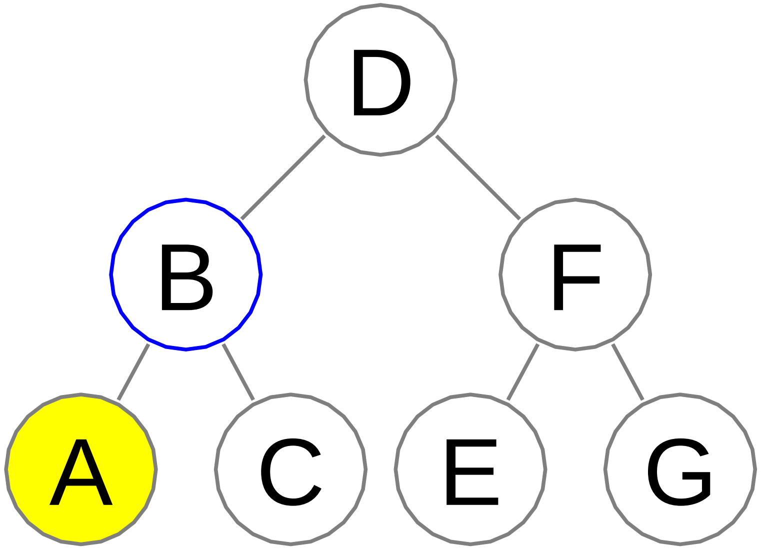
Post-order



Post-order

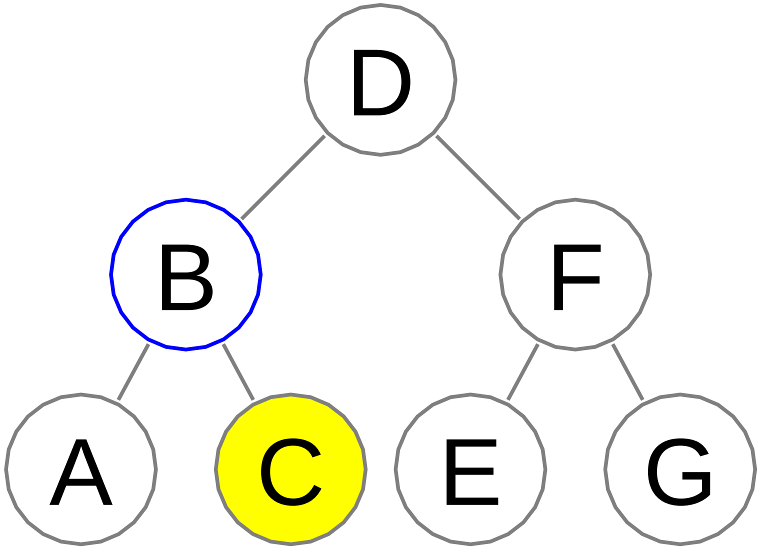


Post-order



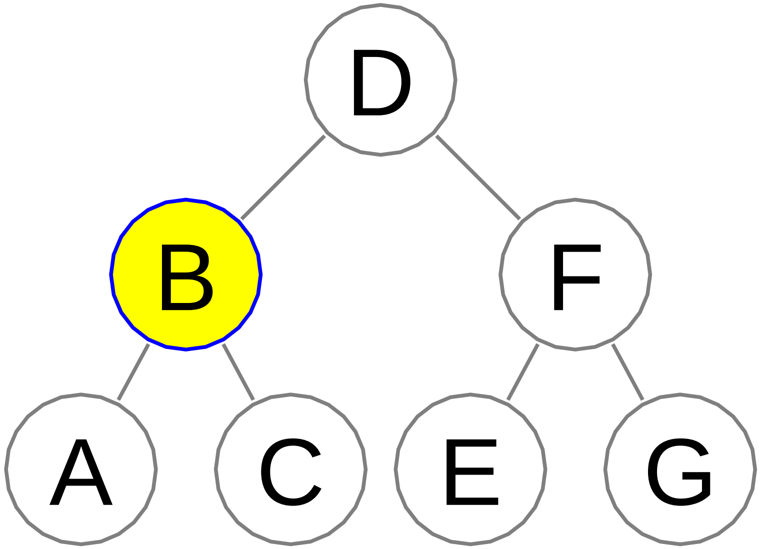
A

Post-order



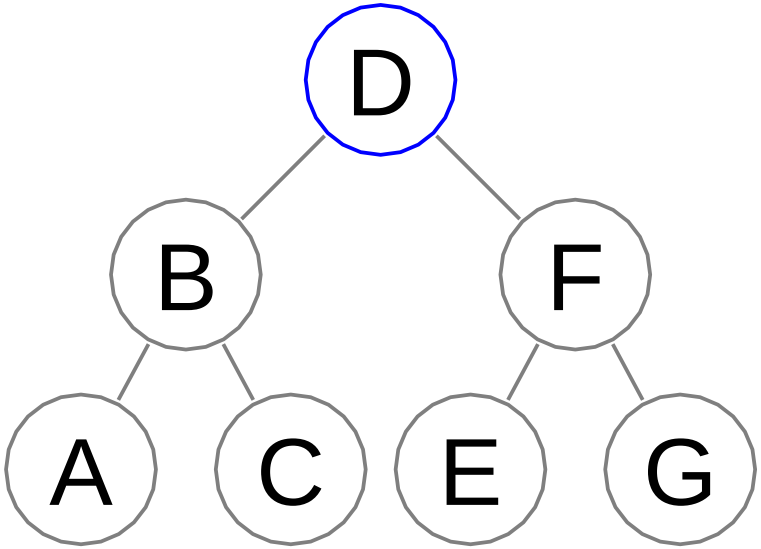
A C

Post-order



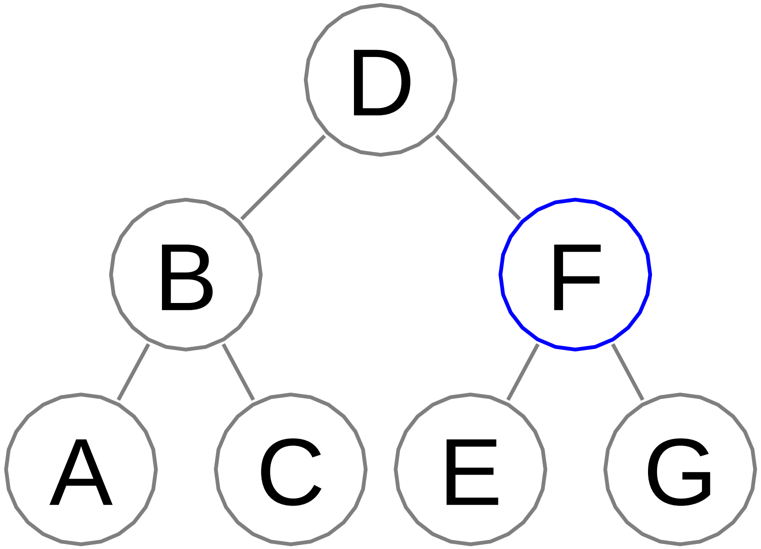
A C B

Post-order



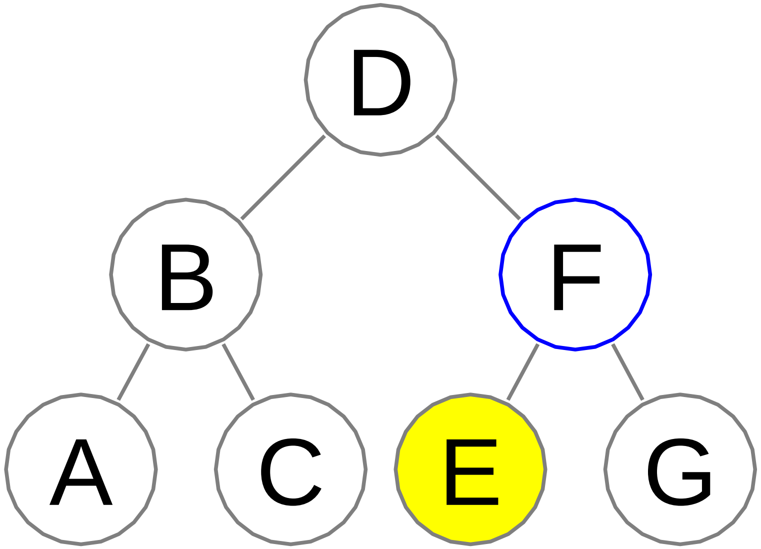
A C B

Post-order



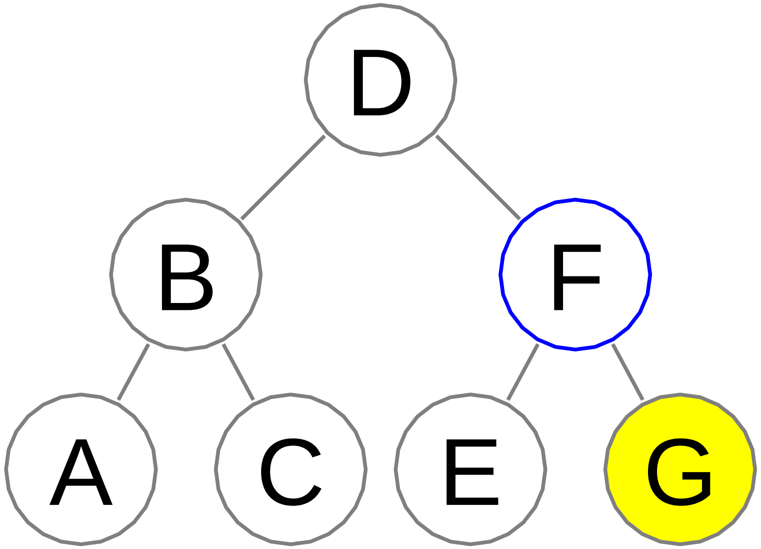
A C B

Post-order



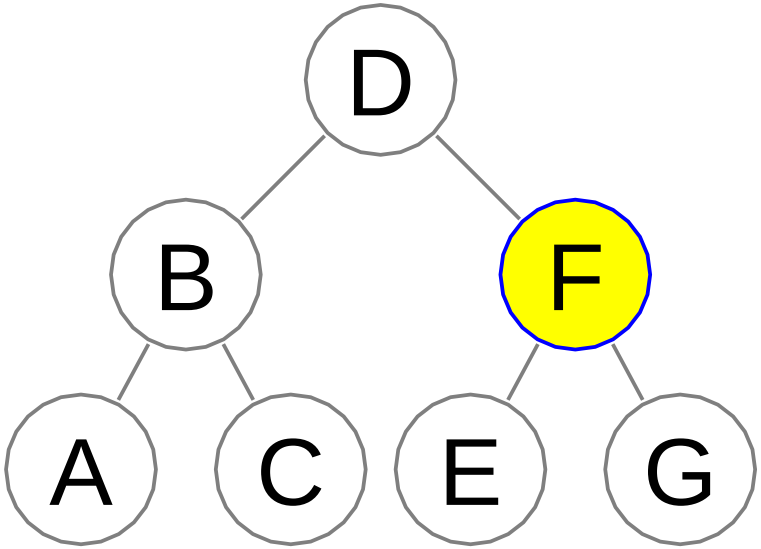
A C B E

Post-order



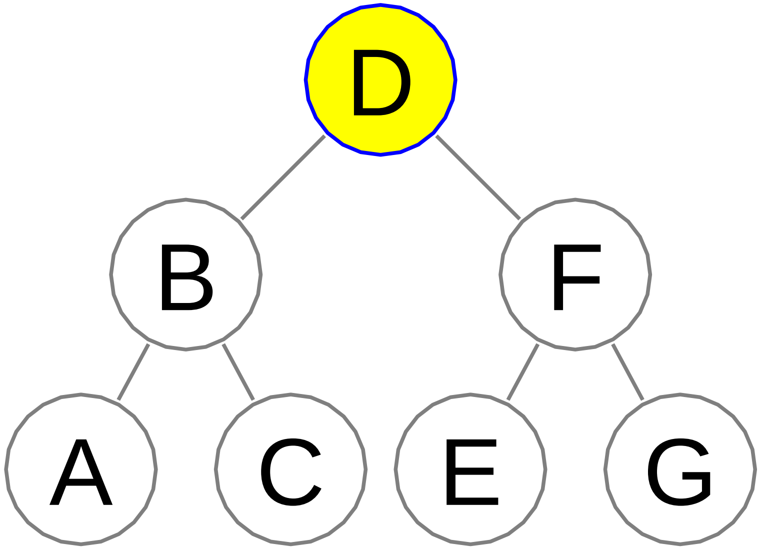
A C B E G

Post-order



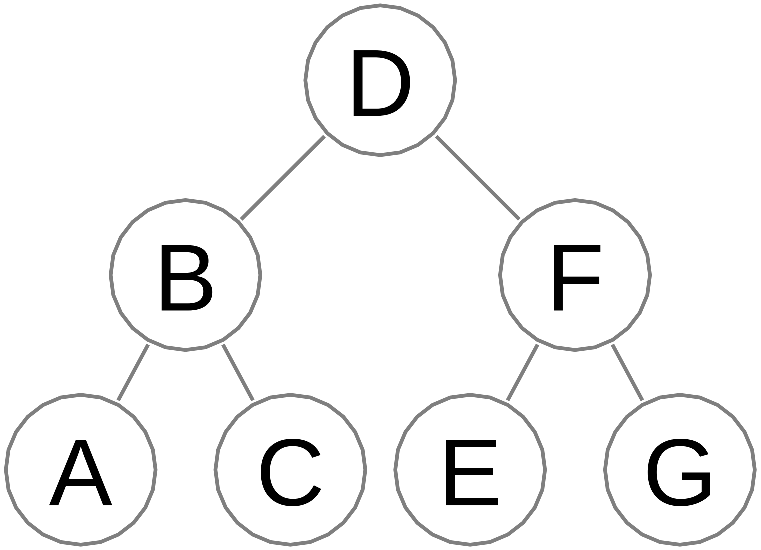
A C B E G F

Post-order



A C B E G F D

Post-order

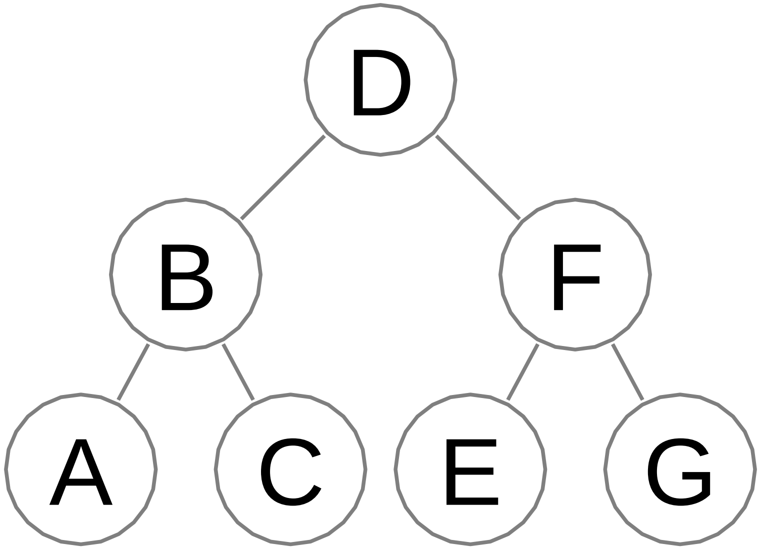


me, node D

A C B E G F D

left subtree of D right subtree of D

Breadth-first traversal



Level traversal:

D

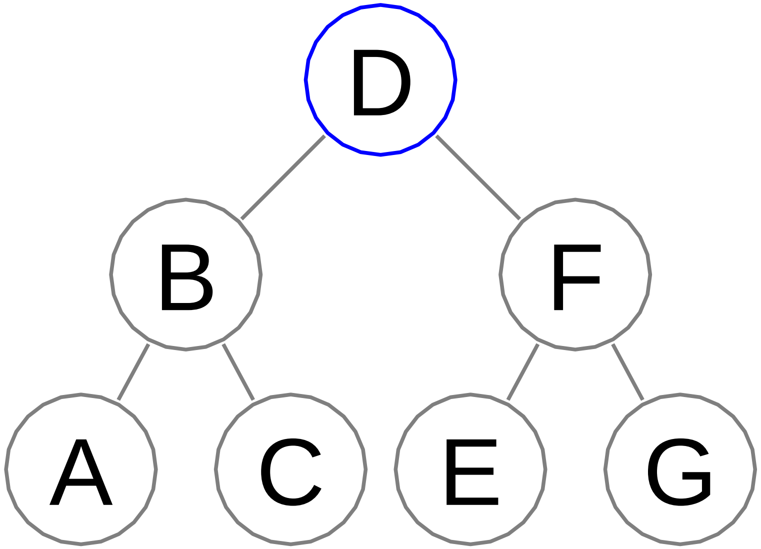
B F

A C E G

Algorithm ***BreadthFirst**Traversal*(*tree*)

```
  if tree == null:
      return
► Queue q
  q.enqueue(tree)
  while not q.isEmpty():
      node ← q.dequeue()
      print(node)
      if node.left != null:
          q.enqueue(node.left)
      if node.right != null:
          q.enqueue(node.right)
```

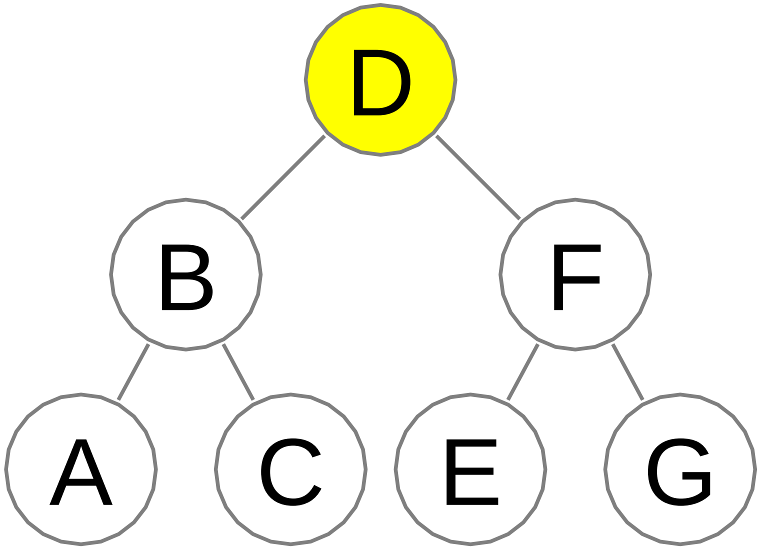
Breadth first: level traversal



Queue: D

Output:

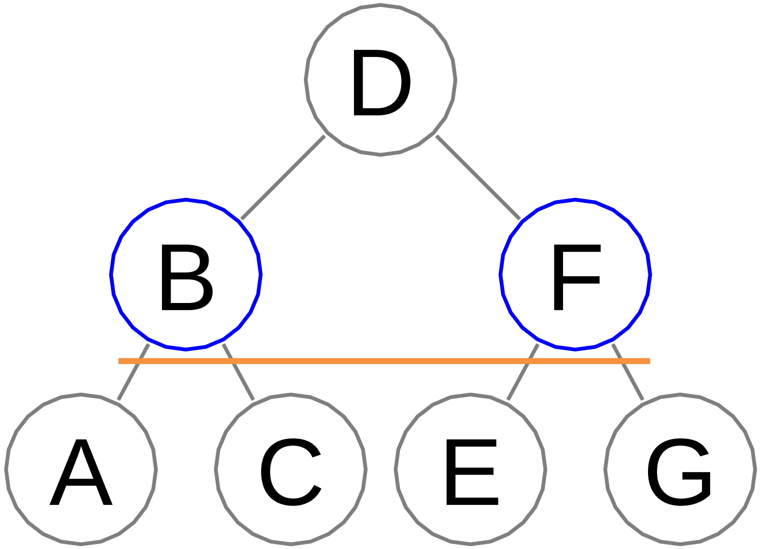
Breadth first: level traversal



Queue:

Output: D

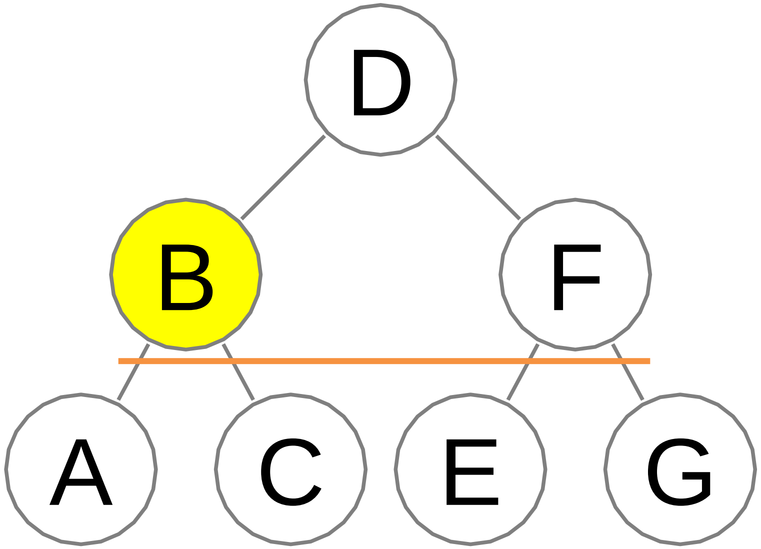
Breadth first: level traversal



Queue: B F

Output: D

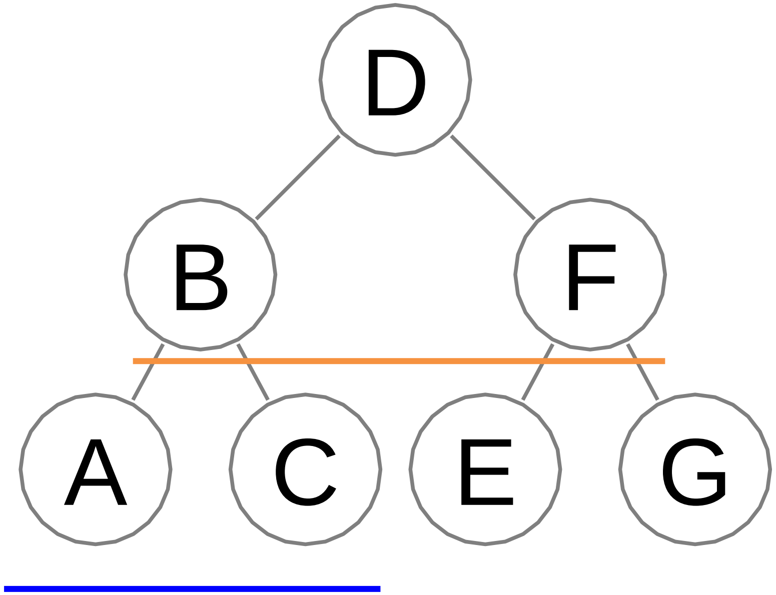
Breadth first: level traversal



Queue: B F

Output: D

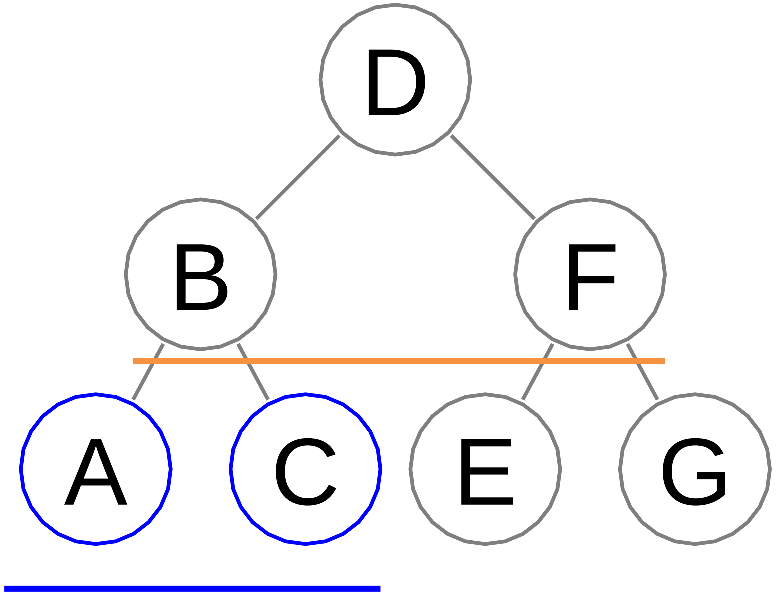
Breadth first: level traversal



Queue: F

Output: D <u>B</u>

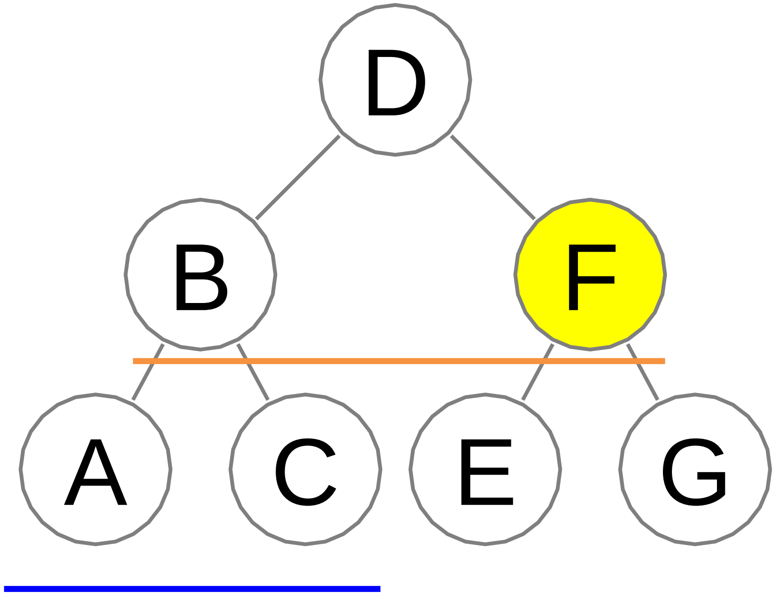
Breadth first: level traversal



Queue: F A C

Output: D B

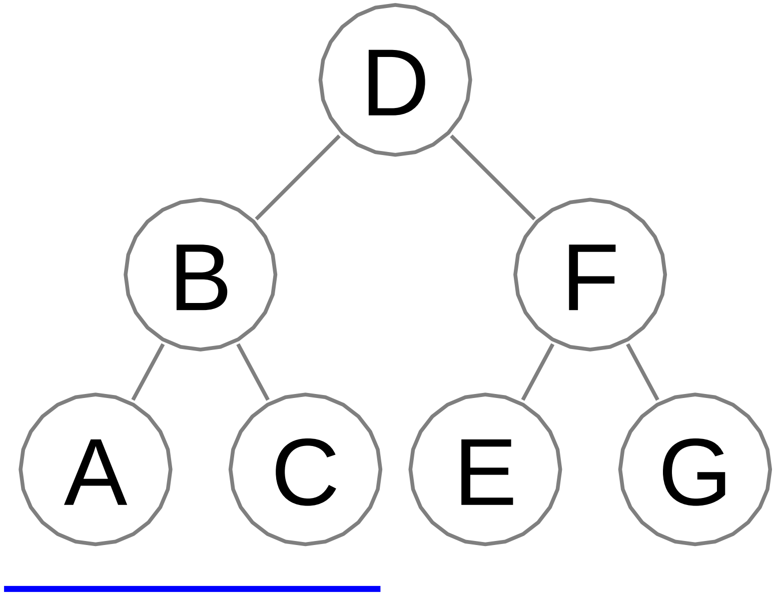
Breadth first: level traversal



Queue: F A C

Output: D B

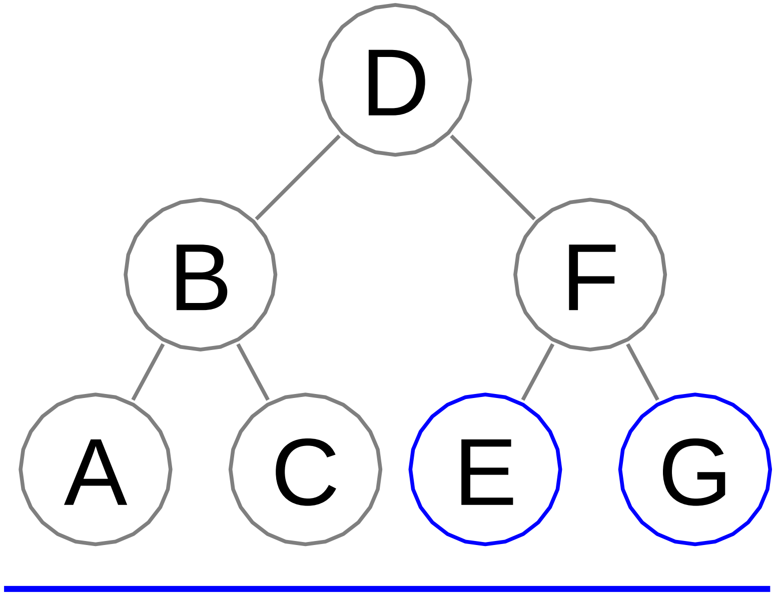
Breadth first: level traversal



Queue: A C

Output: D B F

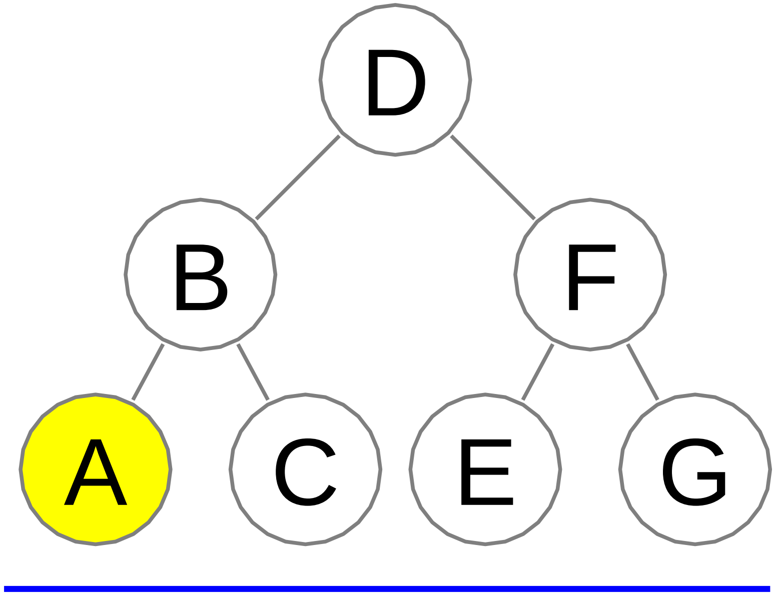
Breadth first: level traversal



Queue: A C E G

Output: D B F

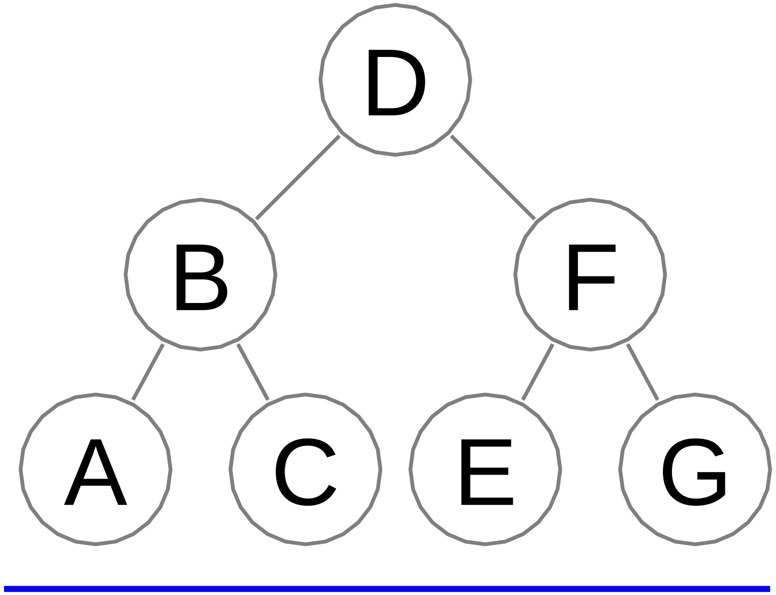
Breadth first: level traversal



Queue: A C E G

Output: D B F

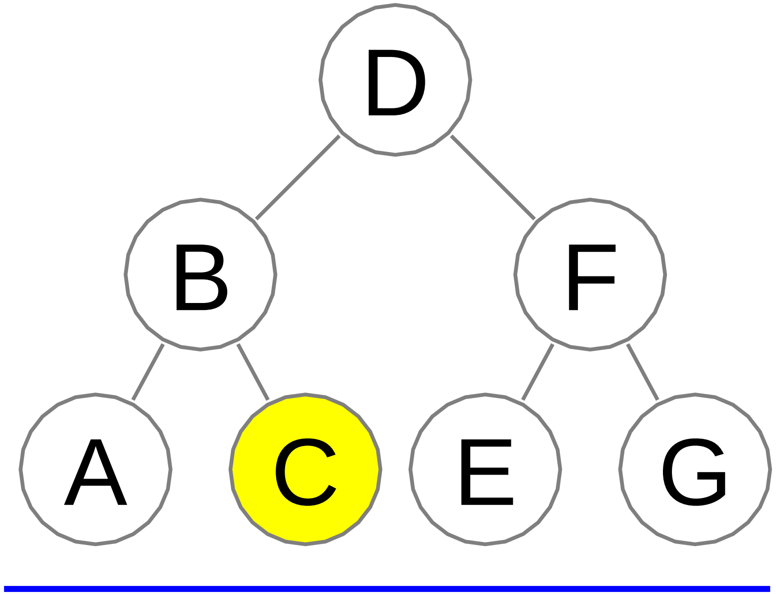
Breadth first: level traversal



Queue: C E G

Output: D B F A

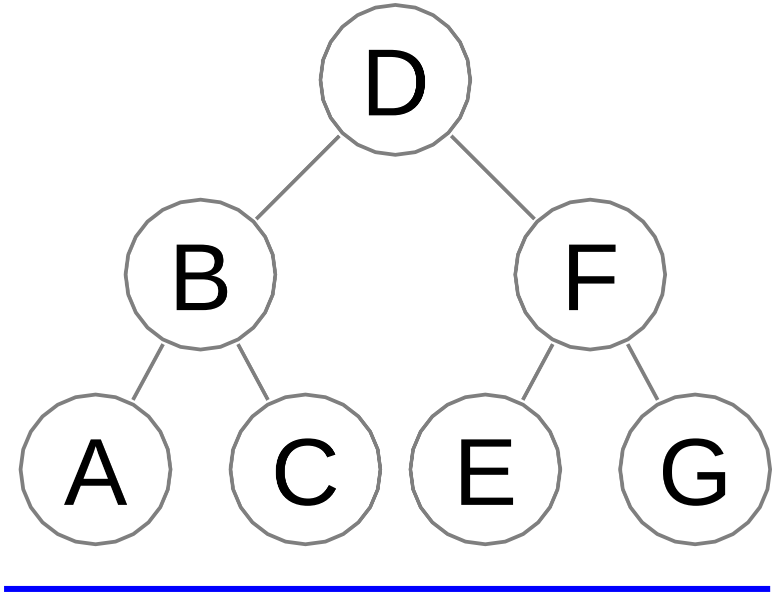
Breadth first: level traversal



Queue: C E G

Output: D B F A

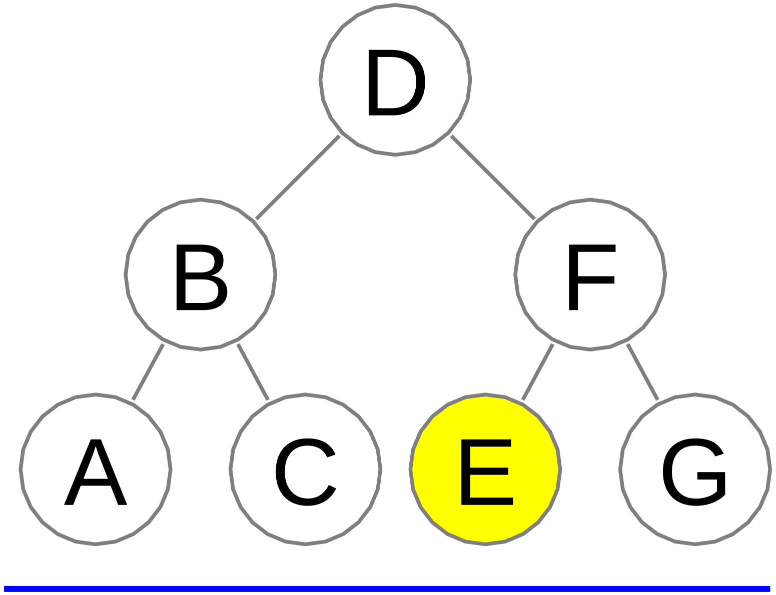
Breadth first: level traversal



Queue: E G

Output: D B F A <u>C</u>

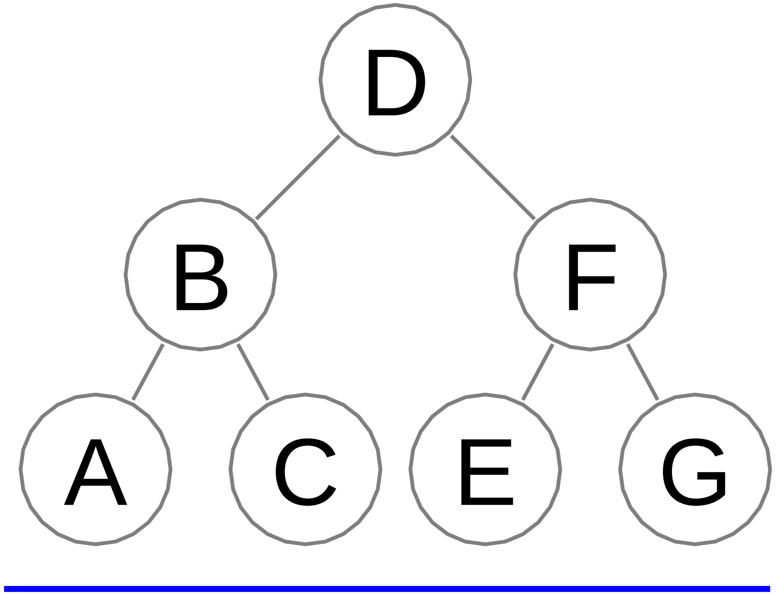
Breadth first: level traversal



Queue: E G

Output: D B F A C

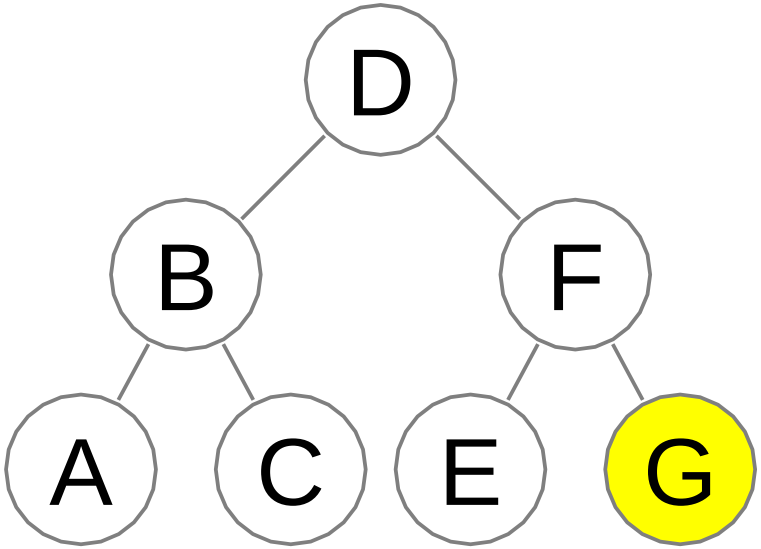
Breadth first: level traversal



Queue: G

Output: D B F A C E

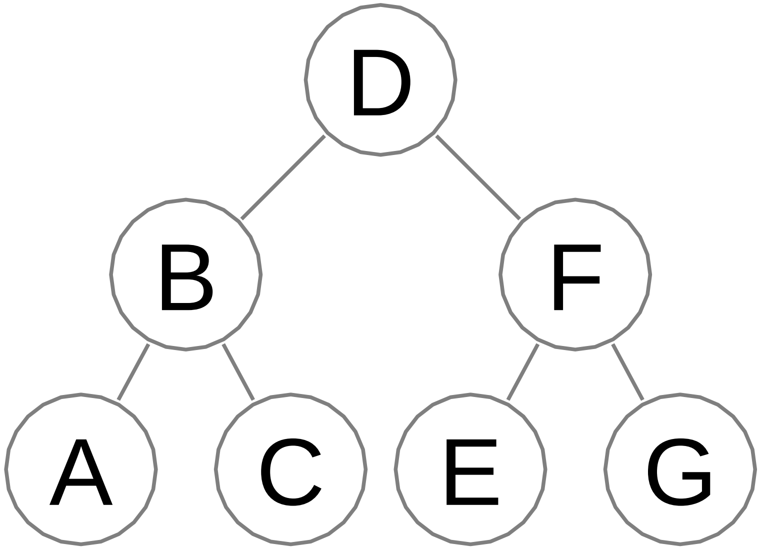
Breadth first: level traversal



Queue: G

Output: D B F A C E

Breadth first: level traversal



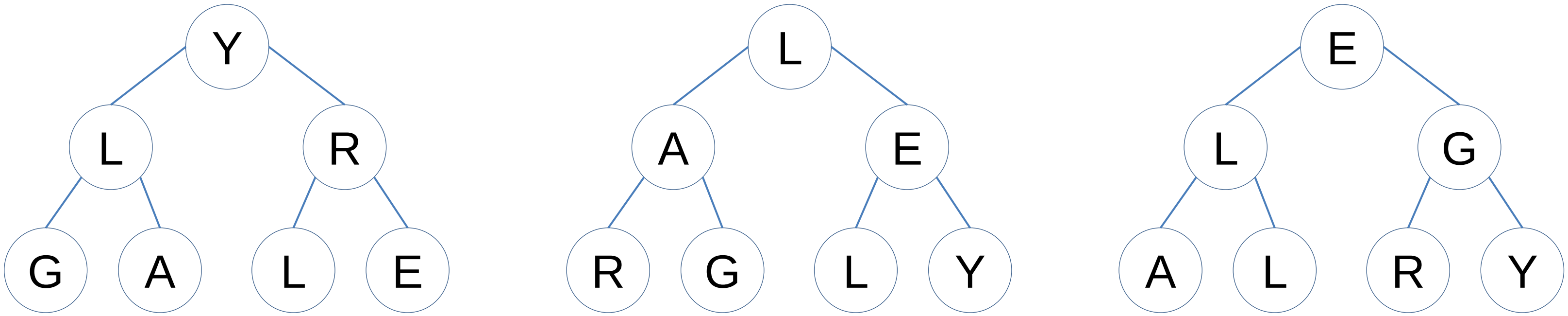
Queue: empty

Output: D B F A C E <u>G</u>

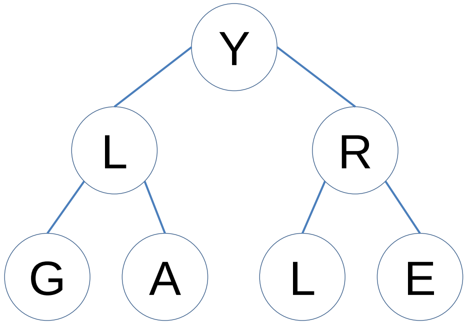
Tree data structure: notes

- *Tree* is fully defined by its root node
- Each node has (at least) a key and links to children
- Tree traversals:
  - Depth-first: uses recursion (stack)
    - pre-order
    - in-order
    - post-order
  - Breadth-first: uses queue
- When working with a tree, recursive algorithms are common
- In Computer Science, trees grow down!

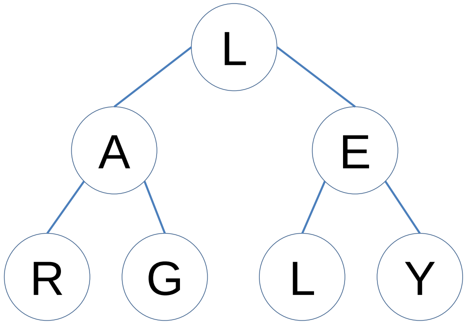
Can you guess which (real) words are spelled by some type of traversal of these trees?



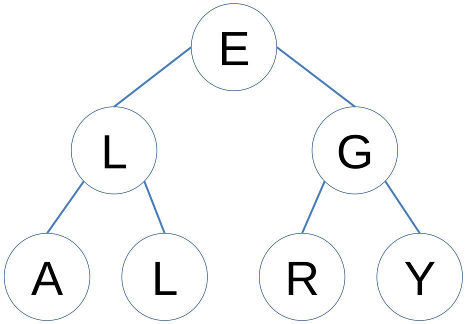
Which type of traversal is used to spell these words?



GALLERY

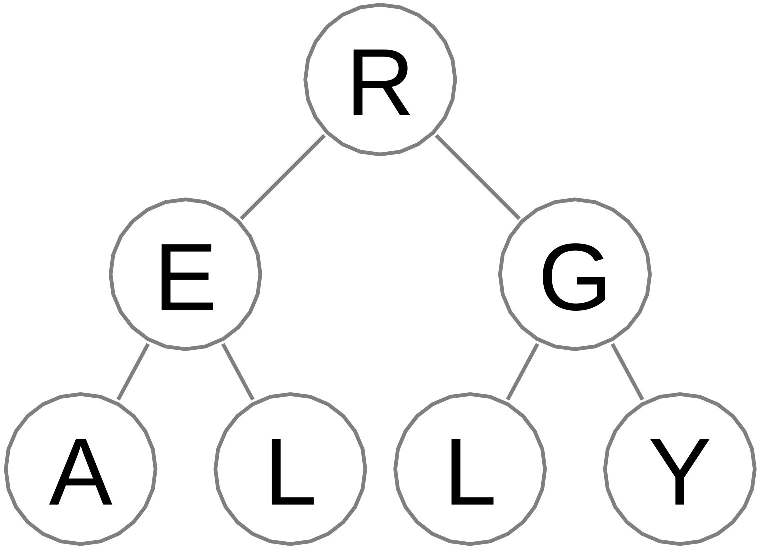


LARGELY

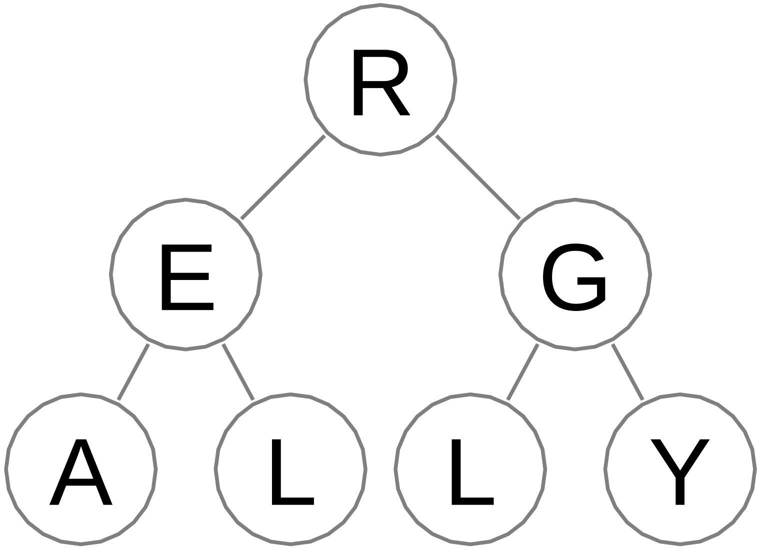


ALLERGY

A. in-order, pre-order, post-order
B. pre-order, post-order, in-order
C. post-order, in-order, pre-order
D. None of the above (something else)



For completeness: **breadth-first** traversal *regally*



For completeness: **breadth-first** traversal